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An analytical study of time-delayed control of friction-induced vibrations in a system with a dynamic friction model

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ABSTRACT

We investigate the control of friction-induced vibrations in a system with a dynamic friction model which accounts for hysteresis in the friction characteristics. Linear time-delayed position feedback applied in a direction normal to the contacting surfaces has been employed for the purpose. Analysis shows that the uncontrolled system loses stability via. a subcritical Hopf bifurcation making it prone to large amplitude vibrations near the stability boundary. Our results show that the controller achieves the dual objective of quenching the vibrations as well as changing the nature of the bifurcation from subcritical to supercritical. Consequently, the controlled system is globally stable in the linearly stable region and yields small amplitude vibrations if the stability boundary is crossed due to changes in operating conditions or system parameters. Criticality curve separating regions on the stability surface corresponding to subcritical and supercritical bifurcations is obtained analytically using the method of multiple scales (MMS). We have also identified a set of control parameters for which the system is stable for lower and higher relative velocities but vibrates for the intermediate ones. However, the bifurcation is always supercritical for these parameters resulting in low amplitude vibrations only.

1. Introduction

Friction-induced vibrations, a type of self-excited oscillations [1,2], are frequently encountered in many engineering systems with sliding components. Some typical examples are brake-squeal [3], train curve squeal [4], clutch chatter [5], machine-tool chatter [6], friction-induced vibrations in robotic joint [7] and lead screw drives [8]. The energy needed for these vibrations comes from a drive system which is in frictional contact with the system of interest. The variable friction force between the contacting surfaces then causes the instability of the driven system. These vibrations should be controlled as they affect the proper operation and performance of these systems. In this paper, we apply time-delayed feedback to control friction-induced vibrations in a system with a dynamic friction model which captures the hysteretic behavior of the friction force frequently observed in experiments. The efficacy of the linear controller applied normal to the friction force in controlling the nature of bifurcation along with quenching these vibrations is also investigated.

There is a vast literature related to research on friction-induced vibrations. The first and the most crucial step in these studies is to identify the instability mechanism which causes these vibrations in the system under consideration. Instability due to the presence of an effective negative damping in the governing equations of motion [9–12] is the mostly studied mechanism in the literature. The negative damping in the system appears due to the drooping characteristic of the friction force in the low relative velocity regime (also known as the Stribeck effect). For systems with multiple degrees of freedom, this negative damping introduces an indefinite damping matrix in the linearized system which is a source of subcritical flutter and squeal [12]. The other two widely studied mechanisms are the mode coupling instability [13–15] and the sprag-slip instability [16,17]. These two instability mechanisms, however, do not require variable friction coefficient to induce vibrations. We will only consider the instability due to the Stribeck effect of the friction force in this paper.

Modeling friction force to describe different experimentally observed friction phenomena is an inherent part of the study of frictional instability. Over the past few decades, several phenomenological friction models with varying degrees of complexity have been proposed in the literature [9–11,20–26]. These friction models, according to their functional forms, can be divided into three categories: (i) static friction models [9,10], (ii) dynamic friction models [21–24] and (iii) acceleration-dependent friction models [25,26]. The friction force for the static friction model depends on the relative velocity of the contacting surfaces only.

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Some typical examples of static friction models are the Coulomb friction model and different Stribeck friction models. These friction models cannot describe the hysteretic effects of friction and so, dynamic friction models wherein the friction force depends on some internal variable and acceleration-dependent friction models with the force depending on the relative acceleration are developed.

Several researchers have experimentally observed the hysteretic or non-reversible characteristics of friction [18,19,25,26] and so, it is an important factor for the complete description of the friction phenomena. In our earlier studies on the control of friction-induced vibrations, we have mainly considered Stribeck friction models [27–29] and hence the results do not correspond fully to practical situations. In this paper, we consider a friction model which is a slight modification of a widely used dynamic friction model (the LuGre model) proposed by Canudas de Wit et al. [22] which can describe the hysteretic effects of friction. Numerical studies show that the bifurcation of the uncontrolled friction-induced system with the LuGre friction model is subcritical in nature [28,30]. The subcritical bifurcation is also sometimes called as hard or dangerous in the engineering literature due to the fact that the stable steady-state near the stability boundary may become unstable due to an unwanted fluctuation of a system parameter resulting in a large amplitude vibration. The supercritical bifurcation is known as soft or safe in a sense that the steady-state is globally stable in the linearly stable region and yields small amplitude vibrations in the unstable region near the stability boundary preventing detrimental damage to the structure. Naturally, supercritical bifurcation is more preferable than the subcritical bifurcation and hence, we also explore controllers which can change the nature of bifurcation to supercritical in this paper as in [28].

We note here that most studies related to friction-induced vibrations consider a mass on a moving belt model or a system similar to this model leading to a single-degree-of-freedom (SDOF) system. Such a model has been used extensively for the study of friction-induced vibrations due to the drooping friction characteristics [9–11] as well as to develop feedback laws to control such vibrations [10,27–35]. There are, however, some researchers who have also considered a two-degree-of-freedom mass on a moving belt model [11,36,37] to study chaotic motions in frictional systems. For simplicity of analysis, we consider a single degree-of-freedom (SDOF) spring–mass–damper system on a moving belt. However, the complexity of the friction force has been retained and the friction model includes hysteresis observed in experiments [18–20].

Researchers have adopted different control strategies (both passive and active) to attenuate or completely quench friction-induced vibrations [10,27–35]. An efficient way to actively control these vibrations is to use time-delayed controllers [27–29,34,35]. A time-delayed controller needs only a position feedback to completely control friction-induced vibrations as opposed to a traditional PD controller which also requires a derivative component. In this paper, we use linear time-delayed position feedback to quench friction-induced vibrations which also enables us to control the nature of the bifurcation. The control force has the same form as discussed in [27–29,35] and is applied in a direction normal to the friction force. We call this control force as the 'normal control force' whose ability to change the nature of the bifurcation comes from the fact that the controller combines with the inherent nonlinearities of the friction force. Preliminary numerical results corresponding to this study were reported in [28,30]. The analysis presented here extends it by obtaining the equations governing the amplitudes of the vibrations using the method of multiple scales which helps in identifying the set of control parameters corresponding to supercritical bifurcations.

The rest of the paper is organised as follows. The mathematical model of the friction-induced system with the time-delayed controller applied normal to the friction force is introduced in Section 2. We perform the linear stability analysis in Section 3 to obtain the stability boundaries separating the linearly stable and unstable equilibria. Non-linear analysis using the method of multiple scales (MMS) is performed in Section 4 to obtain analytical expressions determining the influence of the control parameters on the nature of the bifurcation. Various results are discussed in Section 5. Finally, some conclusions are drawn in Section 6.

2. The mathematical model with a dynamic (LuGre) friction model

A very simple model for the study of friction-induced vibrations is a SDOF spring–mass–damper system with the oscillator mass in frictional contact with a belt moving with a constant velocity. We consider this model with a control force applied along the normal to the contacting surfaces. The physical model is shown schematically in Fig. 1 in which N_0^* is the normal load (including the weight Mg of the oscillator block) in the absence of control force and F_c^* is the control force.

The equation governing the motion of the oscillator is given by

$$M\ddot{X}+C\dot{X}+KX=(N_0^*+F_c^*)f(V_r), \tag{1}$$

where $f(V_r)$ is the friction function (friction force per unit of normal load), $V_r=V_b-\dot{X}$ is the relative velocity and V_b is the belt velocity. The over-dots in Eq. (1) represent derivative with respect to time t. The friction function depends on other internal variables for the dynamic models (to capture the hysteretic behavior) to be considered in this paper and that dependence has not been explicitly written above as these internal variables have not been introduced yet. We consider control force of the following form [27–29,35]:

$$F_c^*=K_c^*(X(t-T^*)-X(t)), \tag{2}$$

where K_c^* is the control gain and T^* is the time-delay. We obtain the non-dimensional equation of motion as

$$x''+2\xi x'+x=(N_0+K_c(x(\tau-T)-x(\tau)))f(v_r). \tag{3}$$

We use the characteristic time scale fixed by $\omega_0=\sqrt{K/M}$ and the characteristic length fixed by $x_0=g/\omega_0^2$ for the non-dimensionalisation. The primes in Eq. (3) denote derivative with respect to the non-dimensional time $\tau=\omega_0 t$. The other non-dimensional quantities are

$$x=\frac{X}{x_0},\quad v_b=\frac{V_b}{\omega_0 x_0},\quad v_r=v_b-x',\quad \xi=\frac{C}{2M\omega_0},$$
$$N_0=\frac{N_0^*}{M\omega_0^2 x_0},\quad K_c=\frac{K_c^*}{M\omega_0^2},\quad T=\omega_0 T^*.$$

The LuGre friction model [22] is used to represent the friction function $f(v_r)$ which is one of the most widely used dynamic friction models to describe the hysteretic effects of friction. In these dynamic models, the friction force does not depend only on the relative velocity but also on some new state variable(s) (often called the internal variable(s)) whose evolution is described by

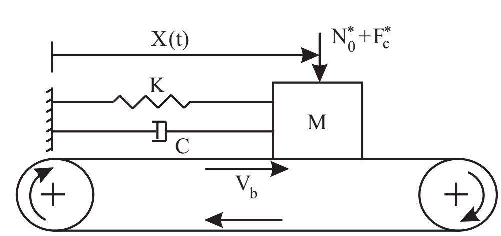

Fig. 1. Damped harmonic oscillator on a moving belt as a model for friction-driven vibrations with normal control force.