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Study of heat transport by stationary magneto-convection in a Newtonian liquid under temperature or gravity modulation using Ginzburg–Landau model

P.G. Siddheshwar [a], B.S. Bhadauria [b,c,*], Pankaj Mishra [c], Atul K. Srivastava [c]

[a] Department of Mathematics, Bangalore University, Bangalore 560001, India
[b] Department of Applied Mathematics and Statistics, School for Physical Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow 226025, India
[c] Department of Mathematics, Faculty of Science, Banaras Hindu University, Varanasi 221005, India

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ABSTRACT

The present paper deals with a weak non-linear stability problem of magneto-convection in an electrically conducting Newtonian fluid, confined between two horizontal surfaces, under a constant vertical magnetic field, and subjected to an imposed time-periodic boundary temperature (ITBT) or gravity modulation (ITGM). In the case of ITBT, the temperature gradient between the walls of the fluid layer consists of a steady part and a time-dependent oscillatory part. The temperature of both walls is modulated in this case. In the problem involving ITGM, the gravity field has two parts: a constant part and an externally imposed time periodic part, which can be realized by oscillating the fluid layer. The disturbance is expanded in terms of power series of amplitude of convection, which is assumed to be small. Using Ginzburg–Landau equation, the effect of modulations on heat transport is analyzed. Effect of various parameters on the heat transport is also discussed.

1. Introduction

There are many interesting situations of practical importance in which the temperature gradient is a function of both space and time. This uniform temperature gradient (temperature modulation) can be determined by solving the energy equation with suitable time-dependent thermal boundary conditions and can be used as an effective mechanism to control the convective flow. Predictions exist for a variety of responses to modulation depending on the relative strength and rate of forcing. Among these, there are the upward or downward shift of convective threshold compared to the unmodulated problems. Lot of work is available in the literature covering how a time-periodic boundary temperature affects the onset of Rayleigh–Bénard convection. An excellent review related to this problem is given by [1].

The effect of temperature modulation on thermal stability in a viscous fluid layer was first considered by Venezian [2]. Performing a linear stability analysis of small amplitude temperature modulation, he derived the onset criteria using a perturbation expansion in powers of the amplitude of oscillation. He showed that the onset of convection can be delayed or advanced depending on the values of the parameters as well as on the types of modulation. Using Galerkin technique and Floquet theory, Rosenblat and Tanaka [3] studied the effect of thermal modulation on the onset of Rayleigh–Bénard convection for rigid–rigid boundaries. The first non-linear problem of thermal instability under temperature modulation was studied by Roppo et al. [4]. They observed that ranges of stable hexagons are produced by the modulation effect near the critical Rayleigh number. Bhadauria and Bhatia [5] studied the effect of temperature modulation on thermal instability by considering rigid–rigid boundaries and different types of temperature profiles. Also Bhadauria [6] studied the effect of temperature modulation respectively, under vertical magnetic field. Further Bhadauria et al. [7] studied the non-linear aspects of thermal instability under temperature modulation, considering various temperature profiles.

The problem of convection in a fluid layer in the presence of complex body forces has gained considerable attention in recent decades due to its promising applications in engineering and technology. The time-dependent gravitational field, one of the complex forces, is of interest in space laboratory experiments, in areas of crystal growth and others. It is also of importance in the large-scale convection in atmosphere. The random fluctuations of gravity field, both in magnitude and direction, experienced in space laboratories, significantly influence natural convection. Existence of adverse density variations within the fluid and a body force are the necessary conditions to initiate natural convection. The idea of using

* Corresponding author at: Department of Mathematics, Faculty of Science, Banaras Hindu University, Varanasi 221005, India.
E-mail addresses: pgsiddheshwar@hotmail.com (P.G. Siddheshwar), mathsbsb@yahoo.com, drbsbhadauria@yahoo.com (B.S. Bhadauria), pankaj1mishra@gmail.com (P. Mishra), atulshaswat@gmail.com (A.K. Srivastava).

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Nomenclature

Latin Symbols

A	amplitude of streamline perturbation
d	height of the fluid layer
g	acceleration due to gravity
k_c	wave number
δ^2	$k_c^2+\pi^2$
Nu	Nusselt number
p	pressure
Pm	magnetic Prandtl number (ν_m/κ_T)
Pr	Prandtl number (ν/κ_T)
Q	Chandarsekhar number ($\mu_m H_b^2 d^2/\rho_0 \nu\nu_m$)
Ra_T	Rayleigh number ($\alpha g \Delta T d^3/\nu\kappa_T$)
R_{0c}	critical Rayleigh number ($\delta^2(\delta^4+Q\pi^2)/k_c^2$)
T	temperature
ΔT	temperature difference across the fluid layer
t	time
x,y,z	space co-ordinates

Greek symbols

α	coefficient of thermal expansion
μ_m	magnetic permeability
ν_m	magnetic viscosity
κ_T	thermal diffusivity
μ	dynamic coefficient of viscosity of the fluid
ν	kinematic viscosity (μ/ρ_0)
ρ	fluid density
ω	Modulation frequency
δ_1	amplitude of gravity modulation
δ_2	amplitude of temperature modulation
ϕ	phase angle
ε	perturbation parameter
Ψ	stream function
Ψ^*	dimensionless stream function
Θ'	temperature perturbation
Θ^*	dimensionless temperature perturbation
Φ	magnetic potential
Φ^*	dimensionless magnetic potential
s	$s=\varepsilon^2 t$ (small time scale)

Other symbols

∇^2	$\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial z^2}$

Subscripts

b	Basic state
$0c$	Critical value
0	Value of the un-modulated case

Superscripts

$'$	perturbed quantity
$*$	dimensionless quantity

mechanical vibration as a tool to improve the heat transfer rate has received much attention.

Gresho and Sani [8] and Greshuni and Zhukhovitskii [9] were the first to study the effect of gravity modulation in a fluid layer. Biringen and Peltier [10] investigated, numerically, the non-linear three dimensional Rayleigh–Bénard problem under gravity modulation, and confirmed the result of Gresho and Sani [8]. Clever et al. [11] performed a detailed non-linear analysis of the problem and presented the stability limits to a much wider region of parameter space. Shu et al. [12] examined the effects of modulation of gravity and thermal gradients on natural convection in a cavity, numerically as well as experimentally. They found that for low Prandtl number fluids, modulations in gravity and temperature produce the same flow field both in structure and in magnitude. Clever et al. [13], Rogers et al. [14], Aniss et al. [15,16], and Bhadauria et al. [17] showed that the gravitational modulation, which can be realized by vertically oscillating a horizontal liquid layer, acts on the entire volume of liquid and may have a stabilizing or destabilizing effect depending on the amplitude and frequency of the forcing. Boulal et al. [18] focused attention on the influence of a quasi-periodic gravitational modulation on the convective instability threshold. They predicted that the threshold of convection corresponds precisely to quasi-periodic solutions.

Magneto-convection arises due to the interaction of electrically conducting fluid flow and the applied magnetic field. Ozoe and Maruo [19] investigated magnetic and gravitational natural convection of metal silicon and performed two-dimensional numerical computations to obtain the rate of heat transfer. Siddheshwar and Pranesh [20] examined the effects of time-periodic temperature/gravity modulation on the onset of magneto-convection in electrically conducting fluids with internal angular momentum by making a linear stability analysis.

In this paper, we use the Ginzburg–Landau (GL) equation to examine non-linear magneto-convection in an electrically conducting Newtonian liquid. GL equations arise as a solvability condition in a wide variety of problems in continuum mechanics while dealing with a weakly non-linear stability of systems.

To the best of authors' knowledge no study is available in which non-linear study of the effect of temperature modulation or gravity modulation on magneto-convection in a Newtonian liquid is considered using Ginzburg–Landau equation. Therefore, in this paper, we study non-linear magneto-convection in the presence of ITBT or ITGM.

2. Mathematical formulation

We consider an electrically conducting liquid of depth d, confined between two infinite, parallel, horizontal planes at $z=0$ and $z=d$. Cartesian co-ordinates have been taken with the origin at the bottom of the liquid layer, and the z-axis vertically upwards. The surfaces are maintained at a constant gradient $\Delta T/d$ and a constant magnetic field $H_b\hat{k}$ is applied across the liquid region (as shown in Fig. 1). Under the Boussinesq approximation, the dimensional governing equations for the study of magneto-convection in an electrically conducting liquid are

$$\frac{\partial \vec{q}}{\partial t}+(\vec{q}\cdot\nabla)\vec{q}=-\frac{1}{\rho_0}\nabla p+\frac{\rho}{\rho_0}\vec{g}+\frac{\mu}{\rho_0}\nabla^2\vec{q}+\frac{\mu_m}{\rho_0}\vec{H}\cdot\nabla\vec{H}, \tag{1}$$

$$\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\kappa_T\nabla^2 T, \tag{2}$$

$$\frac{\partial \vec{H}}{\partial t}+(\vec{q}\cdot\nabla)\vec{H}-(\vec{H}\cdot\nabla)\vec{q}=\nu_m\nabla^2\vec{H}, \tag{3}$$