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Multi-frequency excitation of magnetorheological elastomer-based sandwich beam with conductive skins

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ABSTRACT

The present work deals with the dynamic stability of a symmetric sandwich beam with magnetorheological elastomer (MRE) embedded viscoelastic core and conductive skins subjected to time varying axial force and magnetic field. The conductive skins induce magnetic loads and moments under the application of magnetic field during vibration. The MRE part works in shear mode and hence the dynamic properties of the sandwich beam can be controlled by magnetic fields due to the field dependent shear modulus of MRE material. Considering the core to be incompressible in transverse direction, classical sandwich beam theory has been used along with extended Hamilton's principle and Galarkin's method to derive the governing equation of motion. The resulting equation reduces to that of a multi-frequency parametrically excited system. Second order method of multiple scales has been used to study the stability of the system for simply supported and clamped free sandwich beams. Here the experimentally obtained properties of magnetorheological elastomers based on natural rubber have been considered in the numerical simulation. The results suggest that the stability of the MRE embedded sandwich beam can be improved by using magnetic field.

1. Introduction

Now a days, different materials belong to a class of functionally graded materials and smart materials are used in structures for a wide range of applications and also for active and passive vibration control. Materials with controllable rheological properties, such as magnetorheological elastomers (MRE), electrorheological (ER) and magnetorheological fluids, are being used as semi-active/active vibration devices in various applications such as adaptive tuned vibration absorber (ATVA), variable stiffness suspension bushing, tunable automotive mounts and structures [1–4]. Such materials can provide significant and rapid changes in the damping and stiffness properties. Hoang et al. [5,6] developed new MREs and used it for design of ATVAs for power train vibration reduction. Magnetorheological elastomers comprise of a class of smart materials whose rheological properties can be controlled rapidly and reversibly by an external magnetic field [7–10]. Davis [10] employed the point-dipole model to calculate the shear modulus induced by magnetic field. He observed that when the volume fraction of ferrous particles is 27% the maximum change of the shear modulus occurs. Zhou [11] determined experimentally the field dependent shear storage modulus and the damping factor of the MRE. He showed through a free vibration experiment that the field dependent shear storage modulus could reach a value of 60% of the zero field shear modulus under magnetic field. Later Zhou [12] analyzed the complex shear modulus in three ranges of the frequency domain and reported that in high frequency range the field dependent shear modulus is not affected by the magnetic field.

To improve the MR effect and mechanical performances the natural rubber based MREs have been developed [13,14]. Chen et al. [13] fabricated high modulus MREs based on natural rubber by considering the influences of various fabrication conditions. They reported that the iron particle weight fraction plays an important role in the enhancement of shear modulus. Chen et al [14] developed new MREs, which contains nano-particles of carbon black in addition to iron particles. Experimentally they observed that addition of carbon black in the rubber matrix results in high MR effect, low damping ratio and improved tensile strength.

The MRE have been implemented to achieve controllable properties in sandwich structures with MRE embedded cores between elastic or metal faces. Sandwich beams have been used in aerospace and other applications because of their high strength to weight ratio, low density and good damping capability. Free and forced vibration of sandwich beams with a viscoelastic damping cores have been carried out by many researches. Kerwin [15] developed an expression for a measure of damping of the sandwich beam with a viscoelastic core. Rao [16] obtained frequency and loss factors for various boundary conditions. Free vibration of sandwich beams with viscoelastic cores has been

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studies using classical sandwich beam theory in which the core is assumed to be incompressible in transverse direction [17–20]. Effect of various parameters on the instability regions of a sandwich beam subjected to time varying axial load has been investigated by Ray and Kar [21]. Recently, magnetorheological elastomer (MRE) is embedded in the viscoelastic core to actively attenuate vibration in sandwich structure by applying suitable magnetic field.

The cores with MRE patch possess field controllable flexural rigidity due to the field-dependent shear modulus of the MRE [22–24]. Due to rapid variations in the rheological properties of MRE materials under the application of magnetic field, they are very potential in developing stiffness controllable devices for semi-active and active vibration control of flexible structures [24]. Zhou and Wang [22] analyzed the change of the dynamic flexural rigidity of a MRE embedded soft cored sandwich beam with non-conductive skins under the application of magnetic field. The magnetoelastic loads and its effect on the dynamic properties of the sandwich beam with thin conductive outer skins has been investigated and analyzed by Zhou and Wang [23,24]. A study on flexural rigidities of single layer and multilayer sandwich beams with MRE cores was investigated and effect of various parameters on natural frequencies and efficiency were analyzed on the basis of finite element method [25]. Dwivedy et al. [26] studied the instability regions of a soft cored sandwich beams with MRE patch subjected to periodic axial load using higher order theory. Sun et al. [27] obtained the relationship between the magnetic field and complex shear modulus of the MR materials using oscillatory rheometry techniques. They have also studied experimentally and theoretically the vibration minimization capabilities of the MR adaptive beam. Yalcintas and Dai [28] observed experimentally and theoretically the vibration suppression capabilities of MR adaptive sandwich beams in the form of shifts in natural frequency values, variations in loss factors and vibration amplitudes. Recently, Nayak et al. [29] studied the free and forced vibration of the MRE embedded viscoelastic cored sandwich beam subjected to periodic axial force and static magnetic field. They have investigated the influence of magnetic field on the free vibration response and instability regions of the sandwich beam.

From the above literature, it may be observed that though a significant work has been reported on the dynamic stability analysis of MRE embedded sandwich beam with constant magnetic field (DC component of magnetic field), those for a time varying magnetic field (AC component of magnetic field) is not available. Hence, in this present work an attempt has been made to study dynamic instability of sandwich beam subjected to time varying magnetic field and axial load. The presence of time varying magnetic field and axial load leads the system in the form of a parametrically multi-frequency excited system. The governing equation of this MRE embedded sandwich beam subjected to periodic axial force and periodic magnetic field is developed. Second order method of multiple scales has been used to study the response and stability of the system. MRE containing different percentage of iron particles and nano-sized carbon blacks are considered. Critical parameters of amplitude and frequency of magnetic field and axial load are determined to actively reduce the vibration of the system. Effects of skin thickness, static axial force, static magnetic field, dynamic magnetic field, dynamic axial force, percentage of iron particles (IP) and percentage of carbon black (CB) in attenuation of vibration of the sandwich beam for three resonance conditions are studied.

In this it has been observed that unlike in the system with only DC component of magnetic field where only principal and combination parametric resonances takes places [29], by adding an AC component of the magnetic field additional resonance conditions arises. Hence the system is more prone to large amplitude vibration. Also it has been shown that by adding AC component of magnetic field one may alter the instability region and hence can suppress the vibration of the system by changing the frequency and or amplitude of the AC component of the magnetic field.

2. Modeling

A three layered MRE embedded viscoelastic cored cantilevered sandwich beam of length L, width b, mass per unit length m with two conductive skins is shown in Fig. 1. The thickness of top, core and bottom layers is $2h_t$, $2h_c$ and $2h_b$, respectively. The Young's moduli of top and bottom skin materials are E_t and E_b, respectively. The corresponding moments of inertia are I_t and I_b. G_c is the shear modulus of the core. The time varying magnetic field with flux density $B(t)=B_s+B_d\cos\Omega_1 t$ is applied perpendicular to the skins and parallel to the chain like structures of the ferrous particles inside the MRE. Here B_s is the amplitude of static magnetic field, B_d and Ω_1 are respectively, the amplitude and frequency of the dynamic magnetic field. The beam is subjected to a periodic axial load, $P(t)=P_s+P_d\cos\Omega_2 t$ where P_s is the amplitude of static load, P_d and Ω_2 are respectively the amplitude and frequency of the dynamic axial load. Due to the distributed magnetic field near the skins the magnetoelastic loads are applied to the skins and the bulk dynamic flexural rigidity is affected [24]. The deformed and undeformed cross section of the sandwich beam in this case is similar to that shown in [29, Fig. 2].

Following [29], and using Eqs. (A1)–(A6) for kinetic energy, potential energy and non-conservative work done due to time varying axial load, $P(t)=P_s+P_d\cos\Omega_2 t$ and magnetic field, $B(t)=B_s+B_d\cos\Omega_1 t$ the governing equations of motion are derived by applying the extended Hamilton's principle. The resulting non-dimensional equations of motion for coupled transverse and axial vibrations in terms of longitudinal displacement (u) and transverse displacement (w) are

$$\ddot{\overline{w}}+\frac{1}{mL\omega_s^2}\Bigg[\frac{D}{L^3}\left(1+Y-\frac{B_s^2bD_t^3}{6\mu_eD}-\frac{B_d^2bD_t^3}{12\mu_eD}\right)w_{\overline{xxxx}}-\frac{D}{L^2}\left(\frac{B_s^2bD_t^2}{\pi\mu_0D}\ln\left(\frac{\overline{x}}{1-\overline{x}}\right)\right.$$
$$\left.+\frac{B_d^2bD_t^2}{2\pi\mu_0D}\times\ln\left(\frac{\overline{x}}{1-\overline{x}}\right)\right)\overline{w}_{\overline{xxx}}+\frac{D}{L}\left(\frac{2B_s^2bD_t}{\mu_0D}+\frac{B_d^2bD_t}{\mu_0D}\right)\overline{w}_{\overline{xx}}-\frac{DY}{L^3}\overline{u}_{\overline{xxx}}$$
$$-\frac{D}{L^3}\left(\frac{B_sB_dbD_t^3}{3\mu_eD}\overline{w}_{\overline{xxxx}}+\frac{2B_sB_dD_t^2L}{\pi\mu_0D}\ln\left(\frac{\overline{x}}{1-\overline{x}}\right)\overline{w}_{\overline{xxx}}-\frac{4B_sB_dbD_tL^2}{\mu_0D}\overline{w}_{\overline{xx}}\right)$$
$$\times\cos\overline{\Omega}_1\overline{t}-\frac{D}{L^3}\left(\frac{B_d^2bD_t^3}{12\mu_eD}\overline{w}_{\overline{xxxx}}+\frac{B_d^2bD_t^2L}{2\pi\mu_0D}\right.$$
$$\left.\times\ln\left(\frac{\overline{x}}{1-\overline{x}}\right)\overline{w}_{\overline{xxx}}-\frac{B_d^2bD_tL^2}{\mu_0D}\overline{w}_{\overline{xx}}\right)\cos2\overline{\Omega}_1\overline{t}$$
$$+\frac{D}{L^3}(\overline{P}_s+\overline{P}_d\cos\overline{\Omega}_2\overline{t})w_{\overline{xx}}\Bigg]=0. \tag{1}$$

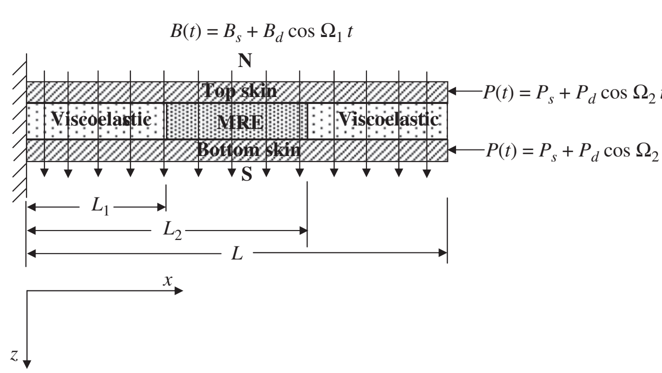

Fig. 1. Schematic diagram of MRE embedded viscoelastic cored sandwich beam subjected to periodic magnetic field and axial load.