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Fast inversion algorithm for identification of elastoplastic properties of power hardening materials from limited spherical indentation tests

Alemdar Hasanov [a,*], Zahir Muradoglu [b]

[a] Department of Mathematics and Computer Sciences, Izmir University, 35350 Izmir, Turkey
[b] Department of Mathematics, Kocaeli University, 41380 Kocaeli, Turkey

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ABSTRACT

An inverse problem of identification of the elastoplastic properties of power hardening engineering materials from limited spherical indentation measurements is studied. A fast algorithm for reconstruction of the Ramberg–Osgood curve $\sigma_i = \sigma_0(e_i/e_0)^\kappa$, with the strain hardening exponent $\kappa \in (0,1)$, is proposed. The main distinguished feature of this algorithm is that the only two output measured data $\langle \alpha_i, \mathcal{P}_i \rangle$, $i=0,1$, i.e. discrete values of the penetration depth (α_i) and the loading force ($\mathcal{P}_i$), are required for the reconstruction of the unknown Ramberg–Osgood curve. The first measured data $\langle \alpha_0, \mathcal{P}_0 \rangle$ corresponds to pure elastic deformations, and the second one to one of the plastic deformations. The second advantage of the proposed algorithm is its well-conditionedness, different from parametrization algorithms proposed in previous studies. Numerical examples related to applicability and enough accuracy of the proposed approach are presented for the noise free and noisy data.

1. Introduction

Spherical indentation testing is one of the extensively used experimental methods to measure material properties from the penetration depth and the loading force curve $\mathcal{P} = \mathcal{P}(\alpha)$ (see [1–3,5,6,8–11,13,14,16–18] and references therein). The idea of relating the mechanical properties of deformable materials to their hardness was first given by Ishlinski [10]. By using the method of characteristics for hyperbolic equations, he has found a relationship between the Brinell hardness $H_B = \mathcal{P}/(2\pi R\alpha)$, and the yield stress: $\sigma_0 = 0.383 H_B$, for a spherically symmetric indentation hardness test (here and below $\mathcal{P} > 0$ is the measured loading force, $R > 0$ is the radius of a spherical indenter and $\alpha > 0$ is the indentation depth). However, in this model the curvature of a contactable surface was ignored, and the problem was considered for a perfectly plastic material. Subsequently, the relationship between hardness as measured data and stress–strain behavior in the regime of "large" indents have been established in [11].

During the last decade, most researches were devoted to the modeling of the spherical indentation process and its numerical simulation. Although it is difficult to give a comprehensive literature survey on this subject, the interested reader can refer to some recent works (see [1–3,8,16–18], and references therein) for numerical simulation and semi-analytical methods. The first important issue here is to construct an appropriate and simple computational inversion method which then can be used in engineering practice. The second important point is that the physical model chosen for simulation of an experiment needs to be adequate from the material behavior point of view. Reformulating gradient plasticity theory, Fleck and Hutchinson [4] proposed generalization of the classical J_2 flow and deformation theories. Results presented in [4] show that, when stressing is nearly proportional (simple loading), as the case of spherical indentation is, the new plasticity models predict qualitatively similar behavior to the J_2 flow and deformation theories. This, in particular, means that J_2-deformation theory of plasticity can be used as a basic physical model in modeling and simulation of a spherical indentation process. On the other hand, as it is shown in the fundamental work of Liu et al. [16], elastoplastic properties of an engineering materials cannot be defined uniquely by using an indentation test, even in within the framework of a simplest physical/mathematical model, in particular, within the J_2-deformation theory of plasticity. The main reason of this phenomenon is the ill-posedness of the inverse problem of determining the unknown stress–strain curve $\sigma_i = \sigma(e_i)$ from the penetration depth–loading force curve $\mathcal{P} = \mathcal{P}(\alpha)$. The mathematical model of this inverse problem has been first proposed in [5]. Note that in view of inverse problems theory for PDEs, non-uniqueness of a solution of an inverse problem is a typical situation in all coefficient identification problems due to their severely ill-posedness (see [8,16,17]).

In engineering literature, identification problems related to spherical indentation and nanoindentation tests have also been analyzed in [1–3,13,14,16–18]. Thus, for near linear hardening

* Corresponding author.
E-mail address: alemdar.hasanoglu@izmir.edu.tr (A. Hasanov).

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materials numerical relationships between material properties and indentation responses have been obtained in [17,18]. An utilization of the connection between the indentation curve $\mathcal{P}=\mathcal{P}(\alpha)$ and other mechanical properties, in particular Brinell Hardness, was given in [10–12]. The main question in all types of identification problems related to determination of elastoplastic properties is the non-uniqueness of a solution. From engineering point of view, the issue of ill-posedness has been analyzed in [13–16]. It was shown in [14,16,17] that indentation test cannot probe material plastic behavior effectively, beyond a critical strain, and hence a solution of the inverse problem of determining the unknown stress–strain curve $\sigma_i=\sigma(e_i)$ from the penetration depth–loading force curve $\mathcal{P}=\mathcal{P}(\alpha)$ is non-unique.

Originally used for hardness measurement, nanoindentation tests are also widely used nowadays for the calibration of various constitutive models. Most often, a nanoindentation curve (load versus penetration) is used to identify the isotropic material parameters with the help of an analytical formula. On the other hand, an inverse analysis of the nanoindentation test can be combined with the additional measured data like residual deformation (imprint) to identify anisotropic constitutive models [2,17].

In this paper, the inverse problem of identification elastoplastic properties of power hardening engineering materials from limited spherical indentation measurements is studied. The indentation is assumed to be frictionless, without unloading. Within the J_2-deformation theory of plasticity, a fast algorithm for reconstruction of the Ramberg–Osgood curve $\sigma_i=\sigma_0(e_i/e_0)^\kappa$ with the strain hardening exponent $\kappa\in(0,1)$ is proposed. Based on computational analysis of the considered inverse problem, limits of ill-posedness are described. In particular, it is shown that even for the considered simple physical model, the non-uniqueness of solution cannot be removed completely. Presented numerical results show that the proposed fast algorithm permits one to obtain a unique and stable reconstruction of the Ramberg–Osgood curve by using only two measured discrete indentation data $\langle\alpha_0,\mathcal{P}_0\rangle$ and $\langle\alpha_1,\mathcal{P}_1\rangle$.

The paper is organized as follows. The formulation of the inverse coefficient problem for uniaxial quasi-static indentation testing is given in Section 2. The finite element discretization of the non-linear direct problem and its linearization are proposed in Section 3. Then the remeshing algorithm is described for generation of synthetic output data for the inverse problem. The fast inversion algorithm and its comparison with the parametrization algorithm are discussed in Section 4. Computational results related to the reconstruction of the Ramberg–Osgood curves from noise free and noisy data are demonstrated in the final Section 5. Appendix A contains derivation of the equilibrium (Lamé) equations corresponding to axisymmetric case from the general equations. In Appendix B an effect of the triangle element geometries on approximation in the proposed remeshing process is discussed.

2. Mathematical model of uniaxial quasi-static indentation and the inverse problem

Let the rigid spherical indenter be loaded with a loading force $\mathcal{P}$, into an axially symmetric homogeneous body, defined to be a sample, occupying the domain $\Omega\times[0,2\pi]$, $\Omega\subset\mathbb{R}^2$, in the negative y-axis direction, as shown in Fig. 1. The uniaxial quasi-static indentation process is simulated by monotonically increasing value $\alpha>0$ of the indentation depth. It is assumed that the indentation process is carried out without unloading, moment and friction. For a given value $\alpha\in(0,\alpha^*)$ of the indentation depth the quasi-static axisymmetric indentation process can be modeled by the following contact problem.

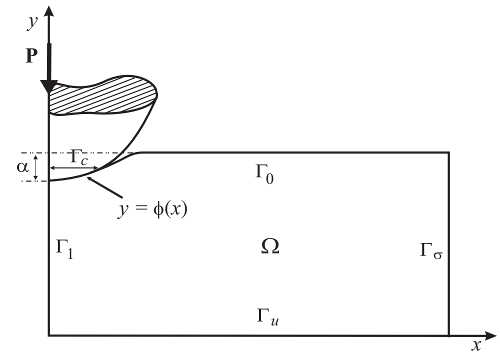

Fig. 1. Geometry of the spherical indentation.

Find the displacement field $u(x,y)=(u_1(x,y),u_2(x,y))$ *from the solution of the unilateral problem*

$$\begin{cases} -\dfrac{\partial}{\partial x}(x\sigma_{11}(u))-\dfrac{\partial}{\partial y}(x\sigma_{12}(u))+\sigma_{33}(u)=0, \\ -\dfrac{\partial}{\partial x}(x\sigma_{12}(u))-\dfrac{\partial}{\partial y}(x\sigma_{22}(u))=0, \quad (x,y)\in\Omega\subset\mathbb{R}^2; \end{cases} \tag{1}$$

$$\begin{cases} \sigma_{11}(u)=0, \quad \sigma_{12}(u)=0, \quad (x,y)\in\Gamma_\sigma; \\ u_1(0,y)=0, \quad \sigma_{12}(u)=0, \quad (x,y)\in\Gamma_1; \\ \sigma_{12}(u)=0, \quad u_2(x,0)=0, \quad (x,y)\in\Gamma_u. \end{cases} \tag{2}$$

$$\begin{cases} u_2(x,l_y)\le-\alpha+\phi(x), \quad \sigma_{22}(u)\le 0, \quad [u_2(x,y)+\alpha-\phi(x)]\sigma_{22}(u)=0, \\ \sigma_{12}(u)=0, \quad (x,y)\in\Gamma_0. \end{cases} \tag{3}$$

Here $\Omega=\{(x,y)\in\mathbb{R}^2: 0<x<l_x, 0<y<l_y\}$, $l_x,l_y>0$, $\Gamma_\sigma=\{(l_x,y): 0<y<l_y\}$, $\Gamma_0=\{(x,l_y): 0\le x\le l_x\}$, $\Gamma_1=\{(0,y): 0<y<l_y\}$, $\Gamma_u=\{(x,0): 0\le x\le l_x\}$, and $\phi(x)=\sqrt{r_0^2-x^2}$ is the surface of the spherical indenter, with the radius $r_0>0$.

Appendix A outlines how the system of Eq. (1) can be derived from the general equation.

The relationship between the components of strain and stress tensors is as follows [12,19,21]:

$$\sigma_{ii}(u)=\tilde{\lambda}\theta(u)+2\tilde{\mu}\varepsilon_{ii}(u), \quad i=1,2,3; \quad \sigma_{12}(u)=2\tilde{\mu}\varepsilon_{12}(u), \tag{4}$$

where $\varepsilon_{11}(u)=\partial u_1/\partial x$, $\varepsilon_{22}(u)=\partial u_2/\partial y$, $\varepsilon_{33}(u)=u_1/x$, $\varepsilon_{12}(u)=0.5(\partial u_1/\partial y+\partial u_2/\partial x)$, $\theta(u)=\varepsilon_{11}(u)+\varepsilon_{22}(u)+\varepsilon_{33}(u)$ the components of deformation, and

$$\begin{aligned} &\tilde{\lambda}=\lambda+2\mu g(e_i^2)/3, \quad \tilde{\mu}=\mu(1-g(e_i^2)), \\ &\lambda=E\nu/[(1+\nu)(1-2\nu)], \quad \mu=E/[2(1+\nu)]. \end{aligned} \tag{5}$$

Here $e_i(u)=(2/3)\{\sum_{i,j=1,3}^{3}[\varepsilon_{ii}(u)-\varepsilon_{jj}(u)]^2+3\varepsilon_{12}^2(u)\}^{1/2}$ is the strain intensity, $\lambda,\mu>0$ are Lamé constants, $E>0$ is an elasticity modulus, $\nu=0.3$ is the Poisson's coefficient and $G=\mu$ is the modulus of rigidity.

The contact problem (1)–(5) for the non-linear system of Lamé equations represents an equilibrium state of an axially symmetric body under the loading force given by the penetration depth $\alpha>0$, in the cylindrical coordinates $(r,z):=(x,y)$.

It is assumed that an axisymmetric sample lies on a substrate without friction, as the last condition in (2) shows. Further, the symmetry of the sample implies the second boundary condition in (2). On the part of the boundary Γ_σ, beyond the contact, the "free boundary conditions" in (2) are given. The contact conditions (3), in the form of inequalities, mean that the contact zone $\Gamma_c(\alpha)=\{(x,l_y)\in\Gamma_0: u_2(x,l_y)= -\alpha+\phi(x), x\in(0,a_c(\alpha))\}, a_c(\alpha):=\partial\Gamma_c(\alpha)$, depending on the value $\alpha>0$ of the indentation depth is also unknown and needs to be defined.