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On rate-dependent dissipation effects in electro-elasticity

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A B S T R A C T

This paper deals with the mathematical modelling of large strain electro-viscoelastic deformations in electro-active polymers. Energy dissipation is assumed to occur due to mechanical viscoelasticity of the polymer as well as due to time-dependent effective polarisation of the material. Additive decomposition of the electric field $\mathbb{E} = \mathbb{E}_e + \mathbb{E}_v$ and multiplicative decomposition of the deformation gradient $\mathbf{F} = \mathbf{F}_e\mathbf{F}_v$ are proposed to model the internal dissipation mechanisms. The theory is illustrated with some numerical examples in the end.

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1. Introduction

Over the past few decades, the theory and the numerical simulation of the coupled electro-mechanical problem have been interesting subjects of research, cf. Pao [1] and Eringen and Maugin [2]. However, with the invention of the so-called electro-active polymers (EAPs) capable of exhibiting large deformations in response to the application of electric fields, several new challenges appear and need to be addressed. Open problems remain both in the understanding of electro-mechanical coupling in soft matter and in simulating the behaviour of electro-sensitive bodies under the influence of an electric field.

EAPs can be used as alternatives to materials traditionally used to develop actuators like piezoelectric ceramics, shape memory metals and electro-rheological fluids, cf. O'Halloran et al. [3]. Potential applications of EAPs in developing artificial muscles and robotic systems include robot manipulators [4], soft pumps [5], loudspeakers [6], portable force feed-back devices [7], haptic interfaces [8], electric generators for energy harvesting [9–11], transport vehicles [12,13], and sensing equipment [14–17], among others.

Efforts were made in the past to model and simulate the behaviour of EAPs using the theory of non-linear elasticity and non-linear visco-elasticity, for example, by Kofod [18], Sommer-Larsen et al. [19], Goulbourne et al. [20], Yang et al. [21,22], and Rosset et al. [23]. However, the papers mentioned above assume that the material electric properties are independent of deformation. Note that because large strain occurs during the deformation process, the non-linearity of the material electric properties must be accounted for. In order to overcome this shortcoming, some related boundary-value problems involving finite deformation were analyzed by taking into account the non-linearity of the electric polarisation, for example, in the works of Voltairas et al. [24], Dorfmann and Ogden [25], Müller et al. [26], Zwecker et al. [27], and Vertechy et al. [28]. The effect of viscosity in the modeling of EAPs was examined recently by Ask et al. [29,30] and Büschel et al. [31].

A basic assumption in the modelling of EAPs in the papers mentioned above has been an instantaneous or 'elastic' response of the material to an applied electric field. This, however, may not be the case in all the electroactive polymers and we aim at modelling this phenomena in this research. We work under a more general case where it is assumed that on the application of an electric field, the overall macroscopic polarisation of the material is time-dependent. Thus, in addition to the mechanical viscoelasticity of the polymeric matrix, an additional energy-dissipating mechanism is considered on account of the evolution of the electric polarisation with time.

Among the several approaches towards a phenomenological theory of mechanical viscoelasticity, the literature is usually divided on account of the nature of internal variable used to quantify dissipation. The internal variable can be assumed to be of stress-type, as proposed by Simo [32] and Lion [33], or it can be strain-type, as used by Lubliner [34], Reese and Govindjee [35] and Huber and Tsakmakis [36]. In the latter approach, which has been also followed in this paper, the deformation gradient is decomposed into elastic and inelastic parts ($\mathbf{F} = \mathbf{F}_e\mathbf{F}_v$) where the inelastic

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part is determined from a differential type flow rule. In addition to the mechanical viscoelastic dissipation, we also model electric dissipation by considering a similar decomposition of our independent variable (electric field in this case) into 'elastic' and 'viscous' parts as $\mathbb{E}=\mathbb{E}_e+\mathbb{E}_v$. This follows a similar approach by Saxena et al. [37] for the magnetic counterpart of this problem. The energy and momentum balance laws of electroelasticity are derived from the fundamental equations of electrostatics following the work of McMeeking and Landis [38].

This paper is organised as follows. The theory of rate-dependent electroelastic deformations is presented in Section 2. Starting with the basic principles of electrostatics and continuum mechanics (as detailed in the Appendix), we obtain the energy and momentum balance laws in the case of electroelasticity. The deformation gradient and the electric field are decomposed into equilibrium and non-equilibrium parts ($\mathbf{F}=\mathbf{F}_e\mathbf{F}_v$, $\mathbb{E}=\mathbb{E}_e+\mathbb{E}_v$). Using the laws of thermodynamics and a form of the free energy density function, constitutive equations are derived along with the conditions to be satisfied by the evolution equations of the non-equilibrium quantities.

For the purpose of obtaining numerical solutions later, the energy density function and the evolution equations for the non-equilibrium quantities are specialised to specific forms. Several electro-visco-elastic coupling parameters are introduced in this step and we define thermodynamically consistent and physically reasonable evolution laws for the internal variables. In Section 3, numerical solutions are obtained corresponding to five different types of (mechanical and electric) loading conditions. The effects of the underlying deformation, strain rate, electric field, and electric field rate are studied on the evolution of the resulting stress and the dielectric displacement. The results, presented graphically, show a strong coupling between strain and electric field, as well as the strong dependence of the response on electro-viscoelastic coupling parameters thus making the model amenable to fitting with experimental data, as and when it becomes available in future.

2. Theory

We consider an electroelastic material that, when undeformed, unstressed and in the absence of electric fields, occupies the material configuration $\mathcal{B}_0$ with boundary $\partial\mathcal{B}_0$. It is then subjected to a static deformation due to the combined action of an electric field, mechanical surface tractions and body forces. The spatial configuration at time t is denoted by $\mathcal{B}_t$ with a boundary $\partial\mathcal{B}_t$. The two configurations are related by a deformation function $\boldsymbol{\chi}$ which maps every point $\mathbf{X}\in\mathcal{B}_0$ to a point $\mathbf{x}=\boldsymbol{\chi}(\mathbf{X},t)\in\mathcal{B}_t$. The deformation gradient is defined as $\mathbf{F}=\text{Grad}\,\boldsymbol{\chi}$, where Grad is the gradient operator with respect to $\mathbf{X}$. Its determinant is given by $J=\det\mathbf{F}$.

2.1. Balance laws and boundary conditions

2.1.1. Equations of electrostatics

Let q be the electric charge density per unit volume in $\mathcal{B}_t$, $\mathbb{e}$ be the spatial electric field vector, $\mathbb{d}$ be the spatial electric displacement vector, and $\mathbb{p}$ be the spatial polarisation vector. The balance equations for the electric quantities are given by a simplified form of the two Maxwell's equations as

$$\text{curl}\,\mathbb{e}=\mathbf{0},\quad \text{div}\,\mathbb{d}=q, \tag{1}$$

where the electric vectors are related by the constitutive law

$$\mathbb{d}=\varepsilon_0\mathbb{e}+\mathbb{p}, \tag{2}$$

and curl and div denote the corresponding differentiation with respect to the position vectors $\mathbf{x}$ in the spatial configuration $\mathcal{B}_t$.

We note that the above equations can also be written in the material configuration $\mathcal{B}_0$ by employing the following transformations:

$$\mathbb{E}=\mathbf{F}^t\mathbb{e},\quad \mathbb{D}=J\mathbf{F}^{-1}\mathbb{d},\quad \mathbb{P}=J\mathbf{F}^{-1}\mathbb{p}, \tag{3}$$

thus giving

$$\text{Curl}\,\mathbb{E}=0,\quad \text{Div}\,\mathbb{D}=Jq,\quad \mathbb{D}=\varepsilon_0 J\mathbf{C}^{-1}\mathbb{E}+\mathbb{P}, \tag{4}$$

such that Curl and Div denote the corresponding differentiation operators with respect to the position vectors $\mathbf{X}$ in $\mathcal{B}_0$ and $\mathbf{C}=\mathbf{F}^t\mathbf{F}$ is the right Cauchy–Green deformation tensor.

Since curl of a gradient vanishes, the electric field vector can be written as the gradient of a scalar potential from Eq. $(1)_1$ as

$$\mathbb{e}=-\text{grad}\,\phi. \tag{5}$$

At an interface or a boundary, the electric vectors must satisfy the conditions

$$\mathbf{n}\times[\![\mathbb{e}]\!]=\mathbf{0},\quad \mathbf{n}\cdot[\![\mathbb{d}]\!]=\hat{q}, \tag{6}$$

where $\hat{q}$ is the surface charge density, $\mathbf{n}$ is the unit outward normal to the surface and $[\![\bullet]\!]$ represents the difference $(\bullet^{\text{out}}-\bullet^{\text{in}})$.

2.1.2. Linear and angular momentum balance

The balance of linear momentum in the configuration $\mathcal{B}_t$ is given in terms of the total Cauchy stress tensor as

$$\text{div}\,\boldsymbol{\sigma}^{\text{tot}}+\mathbf{f}_m=\rho\mathbf{a}. \tag{7}$$

Here $\boldsymbol{\sigma}^{\text{tot}}$ is the total Cauchy stress tensor that takes both mechanical and electric effects into account, $\mathbf{f}_m$ is the purely mechanical body force, ρ is the mass density, $\mathbf{a}$ is the acceleration, and the divergence operator is taken to operate on the first index of a second order tensor. We refer to Appendix A for a detailed derivation of the balance equations in the context of electroelasticity.

The above equation can be written in referential form using the total Piola–Kirchhoff stress $\mathbf{S}^{\text{tot}}=J\mathbf{F}^{-1}\boldsymbol{\sigma}^{\text{tot}}\mathbf{F}^{-t}$ as

$$\text{Div}(\mathbf{S}^{\text{tot}}\mathbf{F}^t)+\mathbf{f}_M=\rho_r\mathbf{a}, \tag{8}$$

with $\rho_r=J\rho$ being the referential mass density and $\mathbf{f}_M=J\mathbf{f}_m$ being the referential body force. Note that the tensor $\mathbf{S}=J\mathbf{F}^{-1}\boldsymbol{\sigma}\mathbf{F}^{-t}$ is sometimes also referred to as the 'second' Piola–Kirchhoff stress.

The principle of balance of angular momentum renders the Cauchy and the Piola–Kirchhoff stress tensors symmetric

$$(\boldsymbol{\sigma}^{\text{tot}})^t=\boldsymbol{\sigma}^{\text{tot}},\quad (\mathbf{S}^{\text{tot}})^t=\mathbf{S}^{\text{tot}}. \tag{9}$$

The corresponding boundary conditions are given by Eqs. (A.5) and (A.6).

2.1.3. Internal dissipation

Very often, the EAPs are synthesised from a rubber like polymer. The polymeric rubber matrix is viscoelastic in nature which leads to energy dissipation on a mechanical deformation. In addition to this, energy dissipation can also occur due to a time-dependent polarisation of the material on application of an electric field. We consider the possibility that on a sudden application of an electric field (or a potential difference), the electric displacement $\mathbb{D}$, the polarisation $\mathbb{P}$, and the resulting electric contribution to stress generated in the material evolve with time to reach an equilibrium value. Thus, the two effects need to be modelled appropriately.

To take into account mechanical viscous effects, we assume the existence of an intermediate configuration $\mathcal{B}_i$ that is, in general, incompatible. The tangent spaces of $\mathcal{B}_0$ and $\mathcal{B}_i$ are related by a second order tensor $\mathbf{F}_v$ that quantifies viscous motion while the tangent spaces of $\mathcal{B}_i$ and $\mathcal{B}_t$ are related by a second order tensor $\mathbf{F}_e$ that quantifies elastic distortion. The configuration $\mathcal{B}_i$ is in parallel to the energy-conserving electroelastic deformation from $\mathcal{B}_0$ to $\mathcal{B}_t$. This motivates the decomposition of the deformation gradient into

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