International Journal of Non-Linear Mechanics 62 (2014) 23–32

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

On the influence of material non-linearities in geometric modeling of kink band instabilities in unidirectional fiber composites

Robert A.E. Zidek, Christina Völlmecke *

Stability and Failure of Functionally Optimized Structures Group, Institute of Mechanics, Technische Universität Berlin, 10587 Berlin, Germany

ARTICLE INFO

Article history:
Received 24 July 2013
Received in revised form
13 January 2014
Accepted 12 February 2014
Available online 20 February 2014

Keywords:
Fiber composites
Compressive failure
Kink banding
Non-linear constitutive law

ABSTRACT

The axial compressive failure of aligned fiber composites triggered by kink band instabilities is the topic of investigation herein. Particular emphasis is put on the accurate prediction of the post-failure regime, where fiber composites are known to exhibit substantial post-failure strength. In this regard, a previous analytical model, based on geometric relationships and energy principles, is enhanced by consistently taking into account material non-linearities. Therefore, a non-linear constitutive law is introduced herein based on a newly developed exponential formulation. This non-linear constitutive law is subsequently implemented into the stress–strain response in interlaminar shearing as well as the compression response. The model enhancements are validated against published experimental data yielding excellent comparisons. Furthermore, the relevance of modeling non-linear material behavior in interlaminar dilation and bending is assessed using a bilinear constitutive law. However, implementing non-linear material behavior does not yield any improvements and can therefore be neglected.

1. Introduction

Owing to their advantageous stiffness and strength-to-weight ratio, fiber composites are increasingly popular for structural applications. Thus, in-depth knowledge about the mechanical properties and failure mechanisms of fiber composites is required. In particular, the longitudinal compressive failure has been subject to extensive research, *e.g.*, [1–8]. The prevailing opinion is that the compressive failure is initiated by local shear buckling of the fibers also referred to as *kinking*. This consequently results in a simultaneous transverse deformation of several layers of the composite, forming a so-called *kink band*. An example of a distinct kink band observed by Vogler and Kyriakides [9] is shown in Fig. 1.

Moreover, the compressive failure of fiber composites is marked by a significant load drop which could potentially lead to catastrophic consequences. However, in several experiments a subsequent load stabilization in the post-failure regime was observed [3,5,10,11]. The load stabilization after compressive failure was likewise confirmed by analytical and numerical kink banding models [3,4,7,8,12].

The accuracy of kink banding models is highly sensitive to the description of the composite's shear response. Early models like the Rosen model [1] assumed linear elastic matrix behavior leading to a substantial overestimation of the compressive failure. Subsequent models by Argon [2] or Budiansky [13] considered non-linear plastic shear behavior. Thereby, the accuracy was significantly improved. Thereafter, the description of non-linear material behavior became more sophisticated. Budiansky and Fleck [14] regarded strain hardening in the shear response by deriving a flow-theory version of plasticity. Likewise, Fleck et al. [4,15] addressed non-linear shearing by using the Ramberg–Osgood description [16]. Similarly, non-linear material behavior plays a decisive role in numerical modeling of kink banding. Vogler et al. [17], for example, compared different material models which particularly affected the response upon initiation of the kink band.

The work presented in this paper mainly focuses on the accurate, analytical prediction of the postbuckling regime with particular attention on the stabilization pressure. Thus, an analytical kink banding model by Wadee et al. [18] is adapted. The aforementioned model is entirely found on geometric relations and energy principles and is initially reviewed herein. As mentioned before, non-linear material behavior, especially in the shear response, significantly influences the mechanical response due to kink banding. For that reason, a constitutive law is developed herein introducing an exponential expression in order to describe the non-linear material behavior of the compound. Thereafter, it is implemented into the model by Wadee et al. [18] in Section 4.

* Corresponding author. Tel.: +49 30 314 22303.
E-mail address: christina.voellmecke@tu-berlin.de (C. Völlmecke).

http://dx.doi.org/10.1016/j.ijnonlinmec.2014.02.004
0020-7462

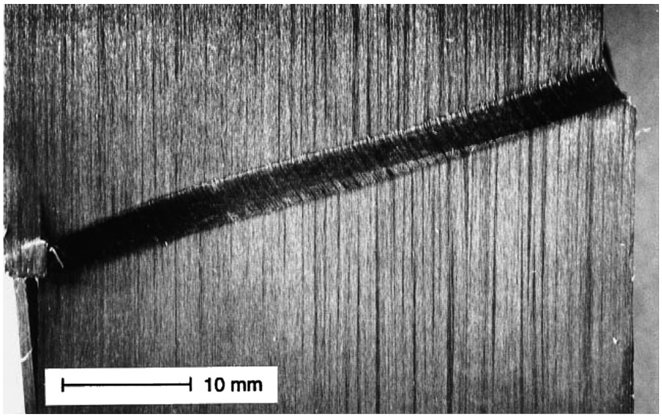

Fig. 1. Photograph of a typical kink band in a fiber composite plate after compressive failure by Vogler and Kyriakides [9].

Moreover, a simplified bilinear constitutive law is utilized to assess the relevance of material non-linearities in bending and interlaminar dilation/compression in Section 5. Results derived using these novel model extensions are validated against experimental data by Kyriakides et al. [3] and significant improvements can be achieved.

2. Review of the precursor model

The precursor model developed by Wadee et al. [18] is purely based on geometric and energy principles and is shown in Fig. 2. The laminated fiber composite of n unidirectional layers is reduced to two adjacent layers within a kink band and is axially compressed by a force nP. The contribution of the remote layers which are unaffected by the kink band is accounted for by linear springs with stiffness k. The transversely isotropic layers are considered homogenous, *i.e.*, the heterogeneous nature of the microstructure is neglected. The shaded region in Fig. 2 illustrates the interlaminar region. The precursor model assumes the laminae to be laterally incompressible and thus the interlaminar region is subject to lateral (orthogonal to the layers) $\delta_{\rm I}$ and shear $\delta_{\rm II}$ displacement. The interlaminar lateral and shear displacements correspond to the forces $F_{\rm I}$ and $F_{\rm II}$, respectively. The rotation of the kinked layers is denoted by α. The bending resistance of the layers is addressed by rotational springs with stiffness c. The kink band width is denoted by b, whereas t stands for a single layer's thickness. Furthermore, the kink band is inclined with respect to the fiber direction by β, which is, however, assumed to be constant during deformation.

The interlaminar displacements can be deduced from geometric relations shown in the enlarged illustration in Fig. 2, thus

$$\delta_{\rm I}(\alpha)=t\left[\frac{\cos(\alpha-\beta)}{\cos(\beta)}-1\right], \tag{1}$$

$$\delta_{\rm II}(\alpha)=\frac{t}{\cos(\beta)}[\sin(\alpha-\beta)+\sin(\beta)]. \tag{2}$$

Consequently, the shear angle γ_{12} reads as

$$\gamma_{12}(\alpha)=\arctan\left(\frac{\delta_{\rm II}}{\delta_{\rm I}+t}\right)=\arctan\left(\frac{\sin(\alpha-\beta)+\sin(\beta)}{\cos(\alpha-\beta)}\right). \tag{3}$$

The equilibrium equations are obtained utilizing the principle of minimum total potential energy. The total potential energy V is the sum of the respective strain energies minus the work done of

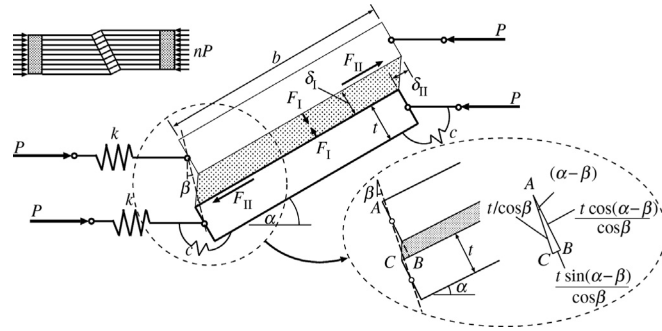

Fig. 2. Kink banding model by Wadee et al. [18].

the external force P along the layer's total end-shortening in longitudinal direction Δ. The total strain energy comprises contributions from interlaminar dilation/compression $U_{\rm D}$, interlaminar shearing $U_{\rm S}$, bending $U_{\rm b}$ and longitudinal compression outside the kink band $U_{\rm m}$. Thus, the total potential energy can be derived as follows:

$$V=U_{\rm D}+U_{\rm S}+U_{\rm b}+U_{\rm m}-P\Delta. \tag{4}$$

The energies $U_{\rm D}$ and $U_{\rm S}$ stored in the interlaminar region are derived by integrating the respective forces $F_{\rm I}$ or $F_{\rm II}$ along the displacement path given by $\delta_{\rm I}$ or $\delta_{\rm II}$, respectively (*cf.* Fig. 2). The obtained expressions are stated in Eq. (3.2) as well as Eqs. (3.6) and (3.8) in [18]. Note that the precursor model assumes a bilinear material response in interlaminar shearing, whereas the remaining energy contributions $U_{\rm D}$, $U_{\rm b}$ and $U_{\rm m}$ are derived according to a linear constitutive law only.

Referring to Fig. 2, the bending energy $U_{\rm b}$ stored in a kinked layer is represented by two rotational springs of stiffness c, thus $U_{\rm b}=c\alpha^2$. An approximation of c is given by Eqs. (3)–(13) in [18]. The axial compression energy $U_{\rm m}$ outside the kink band is addressed by in-line springs with stiffness k. While $\delta_{\rm a}$ is the end-shortening, *i.e.*, the displacement of the in-line springs, the contribution can be derived as $U_{\rm m}=k\delta_{\rm a}^2/2$. The spring's stiffness is defined as $k=E_{11}dt/L$, where L and d are the composite's length and breadth, respectively. E_{11} is the axial Young's modulus of the composite and t is the layer's thickness as stated before.

The equilibrium equations follow from the condition of stationary total potential energy (*cf.* Eq. (4)). Thus, the derivatives of V with respect to the generalized coordinates are to be obtained simultaneously. Herein, these are the end-shortening $\delta_{\rm a}$, the kink band angle α and the kink band width b. Furthermore, the equilibrium equations are non-dimensionalized (denoted by $(\tilde{\ })$) by dividing through kt^2, yielding

$$\tilde{p}=\tilde{\delta}, \tag{5}$$

$$\tilde{p}=\tilde{k}_{\rm I}I_\alpha+\tilde{k}_{\rm II}J_\alpha+\frac{2\tilde{D}\alpha}{\tilde{b}^2\sin(\alpha)}, \tag{6}$$

$$\tilde{p}=\tilde{k}_{\rm I}I_b+\tilde{k}_{\rm II}J_b-\frac{\tilde{D}\alpha^2}{\tilde{b}^2(1-\cos(\alpha))}, \tag{7}$$

where $\tilde{p}$ is the non-dimensional compression stress caused by the force $P{=}pdt$, hence $\tilde{p}=pd/k$. The non-dimensional values $\tilde{\delta}$, $\tilde{k}_{\rm I}$, $\tilde{k}_{\rm II}$ and $\tilde{D}$ are as follows:

$$\tilde{\delta}=\frac{\delta_{\rm a}}{t},\quad \tilde{D}=\frac{L}{12t},\quad \tilde{k}_{\rm I}=\frac{E_{33}L}{E_{11}t},\quad \tilde{k}_{\rm II}=\frac{G_{12}L}{E_{11}t}, \tag{8}$$

where $E_{33}=E_{22}$ is the lateral (or out-of-plane) Young's modulus and G_{12} is the shear modulus of one transversally isotropic unidirectional composite layer. I_α, I_b, J_α and J_b are non-dimensional