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Comparative analysis of chaos control methods: A mechanical system case study

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ABSTRACT

Chaos may be exploited in order to design dynamical systems that may quickly react to some new situation, changing conditions and their response. In this regard, the idea that chaotic behavior may be controlled by small perturbations allows this kind of behavior to be desirable in different applications. This paper presents an overview of chaos control methods classified as follows: OGY methods – include discrete and semi-continuous approaches; multiparameter methods – also include discrete and semi-continuous approaches; and time-delayed feedback methods that are continuous approaches. These methods are employed in order to stabilize some desired UPOs establishing a comparative analysis of all methods. Essentially, a control rule is of concern and each controller needs to follow this rule. Noisy time series is treated establishing a robustness analysis of control methods. The main goal is to present a comparative analysis of the capability of each chaos control method to stabilize a desired UPO.

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1. Introduction

Non-linearities are responsible for a great variety of possibilities in natural systems. Chaos is one of these possibilities being related to an intrinsic richness. A geometrical form to understand chaos is related to a transformation known as Smale horseshoe that establishes a sequence of contraction–expansion–folding which causes the existence of an infinity number of unstable periodic orbits (UPOs) embedded in a chaotic attractor. This set of UPOs constitutes the essential structure of chaos. Besides, chaotic behavior has other important aspects as sensitive dependence to initial conditions and ergodicity.

These aspects of chaos may be exploited in order to design dynamical systems that may quickly react to some new situation, changing conditions and their response. Under this condition, a dynamical system adopting chaotic regimes becomes interesting due to the wide range of potential behaviors being related to a flexible design. The idea that chaotic behavior may be controlled by small perturbations applied in some system parameters allows this kind of behavior to be desirable in different applications.

In brief, chaos control methods may be classified as discrete and continuous methods. Semi-continuous method is a class of discrete method that lies between discrete and continuous method. The pioneer work of Ott et al. [27] introduced the basic idea of chaos control proposing the discrete OGY method. Afterwards, Hübinger et al. [20] proposed a variation of the OGY technique considering semi-continuous actuations in order to improve the original method capacity to stabilize unstable orbits. Pyragas [29] proposed a continuous method that stabilizes UPOs by a feedback perturbation proportional to the difference between the present and a delayed state of the system.

This article deals with a comparative analysis of chaos control methods that are classified as follows: OGY methods – include discrete and semi-continuous approaches [27,20]; multiparameter methods – also include discrete and semi-continuous approaches [10,11]; and time-delayed feedback methods that are continuous approaches [29,34]. Fig. 1 presents an overview of chaos control methods analyzed in this work.

Many research efforts were presented in literature in order to improve the originals chaos control techniques and there are numerous review papers concerning these procedures. In this regard, Shinbrot et al. [33], Ditto et al. [14], Grebogi and Lai [18] and Dubé and Després [15] discussed concepts of chaos and its control presenting discrete chaos control techniques based on OGY method. Pyragas [30] presented an overview of continuous chaos control methods based on time-delayed feedback and mentioned several numerical and experimental applications. Ogorzalek [25], Arecchi et al. [3] and Fradkov and Evans [16] presented review articles that furnish a general overview of chaos control methods, including discrete and continuous techniques. Besides these methods, Boccaletti et al. [6] also treated tracking and synchronization

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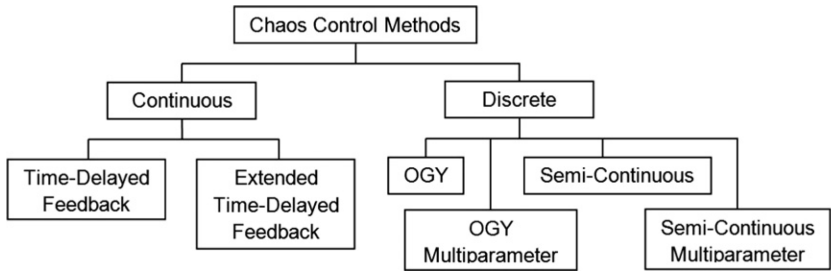

Fig. 1. Chaos control methods.

of chaotic systems and mentioned several experimental implementations. Andrievskii and Fradkov [1] discussed several methods for controlling chaotic systems including chaos control techniques and traditional control methods, while Andrievskii and Fradkov [2] mentioned several works that apply these control procedures to numerous systems of different fields. Fradkov et al. [17] and Savi et al. [32] presented reviews focused on chaos control methods applied to mechanical systems.

Recently, different approaches are being employed in order to stabilize chaotic behavior. In this regard, Kapitaniak [22] applied non-feedback methods by adding a controller, which consists in a linear oscillator, to the dynamical system with the help of coupling elements. Chen [7] presented the design of linear and non-linear conventional feedback controllers based on Lyapunov function methods in other to stabilize chaotic behavior. Bessa et al. [5] proposed an adaptive fuzzy sliding mode strategy enhanced by an adaptive fuzzy algorithm to cope with modeling inaccuracies. The method is applied in order to stabilize UPOs embedded in chaotic response as well as generic orbits.

Despite the numerous review papers concerning the control of chaos, there is a lack of reports that present a comparative analysis of the control strategies, which is the main goal of this contribution. The capability of the chaos control methods to stabilize a desired UPO is analyzed in this paper. A mechanical system is of concern as an application of the general procedure and all signals are generated by numerical integration of a mathematical model, using experimentally identified parameters. In order to consider a system with high instability, a non-linear pendulum treated in other references is considered [11,12,28]. Noise influence is treated by considering signals with observation noise. Results show the performance of each method to stabilize desired orbits exploring some limitations and its application.

The paper is organized as follows. Initially, a brief introduction of chaos control methods is presented. Afterwards, a comparative study is carried out by defining some control rules that should be followed by each controller. Noise influence is treated in the sequence showing the robustness of each controller. Finally, the paper presents the concluding remarks.

2. Chaos control methods

The control of chaos can be treated as a two-stage process. The first stage is called learning stage where the UPOs are identified and system parameters necessary for control purposes are chosen. A good alternative for the UPO identification is the close return method [4]. This identification is not related to the knowledge of the system dynamics details being possible to use time series analysis. The estimation of system parameters is done in different ways for discrete and continuous methods. After the learning stage, the second stage starts promoting the UPO stabilization employing chaos control methods that are discussed in this section.

2.1. OGY method

The OGY method [27] is described by considering a discrete system of the form of a map $\xi^{n+1}=F(\xi^n,p^n)$, where $p\in\Re$ is an accessible parameter for control. This is equivalent to a parameter dependent map associated with a general surface, usually a Poincaré section. Let $\xi_C^{n+1}=F(\xi_C^n,p_0)$ denotes the unstable fixed point on this section corresponding to an unstable periodic orbit in the chaotic attractor that one wants to stabilize. Basically, the control idea is to monitor the system dynamics until the neighborhood of this point is reached. When this happens, a proper small change in the parameter p causes the next state ξ^{n+1} to fall into the stable direction of the fixed point. In order to find the proper variation in the control parameter, δp, it is considered a linearized version of the dynamical system in the neighborhood of the equilibrium point given by Eq. (1). The linearization has a homeomorphism with the non-linear problem that is assured by the Hartman–Grobman theorem [19,36,21,35,31]:

$$\Delta\xi^{n+1}=J^n\Delta\xi^n+w^n\Delta p^n \tag{1}$$

where $\Delta\xi^n=\xi^n-\xi_C^n$, $\Delta\xi^{n+1}=\xi^{n+1}-\xi_C^{n+1}$, and $\Delta p^n=p^n-p_0$. $J^n=D_{\xi^n}\ F(\xi^n,P^n)|_{\xi^n=\xi_C^n,P^n=P_0}$ is the Jacobian matrix and $w^n=D_{p^n}F(\xi^n,\ p^n)|_{\xi^n=\xi_C^n,p^n=p_0}$ is the sensitivity vector.

Fig. 2 presents a schematic picture that allows a geometrical comprehension of the stabilization process. Since the chaotic behavior is related to a saddle point, it is possible to visualize this stabilization over a saddle.

Hübinger et al. [20] verified that the linear mapping J^n deforms a sphere around ξ_C^n into an ellipsoid around ξ_C^{n+1}. Therefore, a singular value decomposition (SVD) can be employed in order to determine the unstable and stable directions, v_u^n and v_s^n, in Σ_n which are mapped onto the largest, $\sigma_u^n u_u^n$, and shortest, $\sigma_s^n u_s^n$, semi-axis of the ellipsoid in Σ_{n+1}, respectively. Here, σ_u^n and σ_s^n are the singular values of J^n:

$$J^n=U^nW^n(V^n)^T=\begin{Bmatrix} u_u^n & u_s^n \end{Bmatrix}\begin{bmatrix} \sigma_u^n & 0 \\ 0 & \sigma_s^n \end{bmatrix}\begin{Bmatrix} v_u^n & v_s^n \end{Bmatrix}^T \tag{2}$$

Korte et al. [23] established the control target as being the adjustment of δp^n such that the direction v_s^{n+1} on the map $n+1$ is obtained, resulting in a maximal shrinking on map $n+2$. Therefore, it demands $\Delta\xi^{n+1}=\alpha v_s^{n+1}$, where $\alpha\in\Re$. Hence

$$J^n\Delta\xi^n+w^n\Delta p^n=\alpha v_s^{n+1} \tag{3}$$

from which α and δp^n can be conveniently chosen.

The OGY method can be employed even in situations where a mathematical model is not available. Under this situation, all parameters can be extracted from time series analysis. The

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