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Non-linear time-dependent flow models of third grade fluids: A conditional symmetry approach

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ABSTRACT

In this communication some non-linear flow problems dealing with the unsteady flow of a third grade fluid in porous half-space are analyzed. A new class of closed-form conditionally invariant solutions for these flow models are constructed by using the conditional or non-classical symmetry approach. All possible non-classical symmetries of the model equations are obtained and various new classically invariant solutions have been constructed. The solutions are valid for a half-space and also satisfy the physically relevant initial and the boundary conditions.

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1. Introduction

During the past several decades, the study of non-linear problems dealing with flow models of non-Newtonian fluids has gained prodigious attention. This interest is due to several important applications in engineering and industry such as aerodynamic heating, electrostatic precipitation, petroleum industry, reactive polymer flows in heterogeneous porous media, electrochemical generation of elemental bromine in porous electrode systems, manufacture of intumescent paints for fire safety applications, extraction of crude oil from petroleum products, synthetic fibers, paper production and so forth. Due to the intricate microstructure of non-Newtonian fluids, there is no single constitutive expression available in the literature which describes the physical behavior and properties of all non-Newtonian fluid models. Because of this, several models of non-Newtonian fluids have been proposed. The mathematical modeling of non-Newtonian incompressible fluid flows gives rise to complicated non-linear differential equations. The situation becomes more involved when we consider exact (closed-form) solutions of these problems. Several techniques and methods have been developed recently to construct solutions of non-Newtonian fluid flow problems. Some of these useful methods are the variational iteration method, Adomian decomposition method, homotopy perturbation method, homotopy analysis method, semi-inverse variational method and symmetry method. Despite all these methods, exact (closed-form) solutions of non-Newtonian fluid flow problems are still rare in the literature and it is not surprising that new exact (closed-form) solutions are most welcome provided they correspond to physically realistic situations.

Amongst the many models which have been used to describe the physical flow behavior of non-Newtonian fluids, the fluids of differential type have received special attention as well as much controversy, see for example [1] for a complete discussion of the relevant issues. Rivlin–Ericksen fluids of differential type have secured special attention in order to describe several non-standard characteristics of non-Newtonian fluids such as rod climbing, shear thinning, shear thickening and normal stress effects. Literature surveys point out that much focus has been given to the flow problems of a non-Newtonian second grade fluid model [2]. A second grade fluid model is the simplest subclass of differential type fluids for which one can reasonably hope to establish an analytic result. In most of the flow aspects, the governing equations for a second grade fluid are linear. Although a second grade fluid model for steady flows is used to predict the normal stress differences, it does not correspond to shear thinning or thickening if the shear viscosity is assumed to be constant. Therefore some experiments may be better described by a third grade fluid. The mathematical model of a third grade fluid represents a more realistic description of the behavior of non-Newtonian fluids. A third grade fluid model represents a further attempt toward the study of the flow structure of non-Newtonian fluids. Therefore, a third grade fluid model is considered in this study. This model is known to capture the non-Newtonian effects such as shear

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thinning or shear thickening as well as normal stress. The governing equations for the third grade fluid model are non-linear and much more complicated than those of Newtonian fluids. They require additional boundary conditions to obtain a physically meaningful solution. This issue has been discussed in detail by Rajagopal [3,4] and Rajagopal and Kaloni [5]. Fosdick and Rajagopal [6] made a complete thermodynamical analysis of a third grade fluid and derived the restriction on the stress constitutive equation. They investigated some stability characteristics of third grade fluids and showed that they exhibit features different from those of Newtonian and second grade fluids. Further, one can refer to the important works of Ariel [7], Akyildiz [8,9], Sahoo [10], Aksoy and Pakdemirli [11], Makinde [12], Passerini and Patria [13], Mollica and Rajagopal [14], Rajagopal and Na [15] and Rajagopal [16] regarding the flow of a third grade fluid.

The present paper continues the research which was carried out in [17–19]. In Hayat et al. [17], some classically invariant solutions were obtained for unsteady flow of a third grade fluid in a porous medium. In [18], the analysis of [17] was extended by taking into account the magnetohydrodynamic (MHD) nature of the fluid. We also utilized the Lie classical symmetry approach to construct a new class of group invariant solutions for the governing non-linear partial differential equation. Aziz and Aziz [19] recently extended the flow model of [18] by including suction and injection at the boundary of the flow. The same group theoretic approach has been used to obtain closed-form invariant solutions of the non-linear boundary-value problem. In this work, we revisit these three flow problems. A conditional symmetry approach is employed to construct some new exact solutions of these models. The conditional symmetry or non-classical symmetry approach has its origin in the work of Bluman and Cole [20]. In recent years, interest in the conditional symmetry approach has increased. Information on the non-classical method and related topics can be found in [21–24]. This method has also been used to obtain new exact solutions of a number of interesting non-linear partial differential equations [25–30]. There are equations arising in applications that do not admit classical symmetries but have conditional symmetries. Thus this method is useful in constructing exact solutions. The concept of conditional/non-classical symmetry has not been used to find conditionally-invariant solutions of non-Newtonian fluid flow problems. This is the first time that the conditional symmetry approach has been employed to tackle non-linear problems dealing with the flow models of non-Newtonian third grade fluids. We believe that these deserve further importance in tackling non-Newtonian fluid flow problems, which are the main foci of this paper. A summary of the non-classical symmetry method is presented in the Appendix.

2. Problem 1: Unsteady flow of a third grade fluid over a flat rigid plate with porous medium

Hayat et al. [17] solved the time-dependent problem for the flow of a third grade fluid in a porous half-space. The governing non-linear partial differential equation in [17], with a slight change of notation, is given by

$$\rho\frac{\partial u}{\partial t}=\mu\frac{\partial^2 u}{\partial y^2}+\alpha_1\frac{\partial^3 u}{\partial y^2\partial t}+6\beta_3\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-\frac{\phi}{\kappa}\left[\mu+\alpha_1\frac{\partial}{\partial t}+2\beta_3\left(\frac{\partial u}{\partial y}\right)^2\right]u, \tag{1}$$

where $u(y,t)$ is the velocity component, ρ is the density, μ the coefficient of viscosity, α_1 and β_3 are the material constants (for details on these material constants and the conditions that are satisfied by these constants, the reader is referred to [6]), ϕ the porosity and κ the permeability of the porous medium.

In order to solve the above Eq. (1), the relevant time and space dependent velocity boundary conditions are specified as follows:

$$u(0,t)=u_0V(t),\quad t>0, \tag{2}$$

$$u(\infty,t)=0,\quad t>0, \tag{3}$$

$$u(y,0)=g(y),\quad y>0, \tag{4}$$

where u_0 is the reference velocity. The first boundary condition (2) is the no-slip condition and the second boundary condition (3) says that the main stream velocity is zero. This is not a restrictive assumption since we can always measure velocity relative to the main stream. The initial condition (4) indicates that the fluid is initially moving with some non-uniform velocity $g(y)$.

In [17], the governing problem was solved without transforming the problem into dimensionless form. Here we first non-dimensionalize the problem and then obtain solutions. Defining the non-dimensional quantities as

$$\overline{u}=\frac{u}{u_0},\quad \overline{y}=\frac{u_0y}{\nu},\quad \overline{t}=\frac{u_0^2t}{\nu},\quad \overline{\alpha}=\frac{\alpha_1u_0^2}{\rho\nu^2},\quad \overline{\beta}=\frac{2\beta_3u_0^4}{\rho\nu^3},\quad \frac{1}{\overline{K}}=\frac{\phi\nu^2}{\kappa u_0^2}. \tag{5}$$

Eq. (1) and the corresponding initial and the boundary conditions take the form

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+\alpha\frac{\partial^3 u}{\partial y^2\partial t}+3\beta\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-\frac{1}{K}\left[u+\alpha\frac{\partial u}{\partial t}+\beta u\left(\frac{\partial u}{\partial y}\right)^2\right], \tag{6}$$

$$u(0,t)=V(t),\quad t>0, \tag{7}$$

$$u(y,t)\to 0\quad \text{as } y\to\infty, t>0, \tag{8}$$

$$u(y,0)=g(y),\quad y>0, \tag{9}$$

where $V(0)=g(0)$. For simplicity we suppress the bars of the non-dimensional quantities. We rewrite Eq. (6) as

$$\frac{\partial u}{\partial t}=\mu_*\frac{\partial^2 u}{\partial y^2}+\alpha_*\frac{\partial^3 u}{\partial y^2\partial t}+\beta\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-\beta_*u\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{K_*}u, \tag{10}$$

where

$$\mu_*=\frac{1}{(1+\alpha/K)},\quad \alpha_*=\frac{\alpha}{(1+\alpha/K)},\quad \beta=\frac{3\beta}{(1+\alpha/K)},$$
$$\beta_*=\frac{\beta/K}{(1+\alpha/K)},\quad \frac{1}{K_*}=\frac{1/K}{(1+\alpha/K)}. \tag{11}$$

We solve Eq. (10) subject to the conditions (7)–(9).

2.1. Non-classical symmetry analysis

Here we present the complete non-classical symmetry analysis of PDE (10) and develop some classically invariant solution of Problem 1.

Consider the infinitesimal operator

$$\chi=\xi^1(t,y,u)\frac{\partial}{\partial t}+\xi^2(t,y,u)\frac{\partial}{\partial y}+\eta(t,y,u)\frac{\partial}{\partial u}. \tag{12}$$

The invariant surface condition is

$$\phi(t,y,u)=\eta(t,y,u)-\xi^1(t,y,u)\frac{\partial u}{\partial t}-\xi^2(t,y,u)\frac{\partial u}{\partial y}=0. \tag{13}$$

The non-classical symmetries determining equations are

$$\chi^{[3]}\text{Eq.(10)}|_{\text{Eq.(10)}=0,\phi=0}=0, \tag{14}$$

where $\chi^{[3]}$ is the usual third prolongation of operator χ.

Here we find the determining equations for two different cases:

Case 1. When $\xi^1=1$ and $\xi^2\neq 0$

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