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Analytical evaluation of the elastic and plastic resistances of double symmetric rectangular hollow sections under axial force and biaxial bending

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ABSTRACT

In the recent codes for the design of steel structures, the elastic–plastic methods of analysis are recognised to provide an efficient estimation of the ultimate resistance of some of these structures. These methods are usually based on some basic hypotheses, such as the creation of plastic hinges in the most stressed cross-sections, for instance.

As the development of these plastic hinges depends on the interaction between the internal forces and on the cross-section shape, specific equations are required for the analysis of different types of cross-sections. However, most frequently, these equations are not available, or they are expressed by means of simplified expressions; this is usually the case when biaxial bending is involved.

This paper presents new interaction criteria for the analysis of steel rectangular hollow sections subjected to an axial force and biaxial bending moments, at the elastic or the plastic limit states (as long as buckling phenomena are not involved). The plastic interaction criteria are presented, in a first step, for some particular combinations of the internal forces, such as axial loading with bending about a main axis, and biaxial bending without axial loading. Then, the global solution for the simultaneous combination of an axial force and bending moments about both the main axes of inertia are described in detail. All these plastic interaction criteria are compared with the corresponding plastic criteria adopted in the Eurocode 3 (EC3). Some suggestions are presented in order to improve the results given by these EC3 criteria.

1. Introduction

The analysis of the behaviour and limit carrying capacity of a cross-section under biaxial bending is usually a complex problem, which has been studied by many researchers for a long time. A large number of publications may be found, covering the study of structural cross-sections made of different materials (such as reinforced concrete sections [12,27], composite steel–concrete sections [23,24], steel sections [29], or aluminium sections [11], for instance). A review of different methods used for the evaluation of the cross-sections plastic resistance may be found in [25]; most of them essentially consider only axial stresses for the determination of the plastic section capacity. Shear stresses from uniform torsion, warping torsion and from shear forces are either disregarded or considered only approximately in some of those approaches [25].

In the case of steel sections, a considerable amount of research has been done concerning the study of different types of cross-sections, such as H and I shapes [18], solid and hollow rectangular sections [16,29], or angle sections [31,32]. Some extensive reviews of these research works may be found in several publications, such as [13] or [18] for instance.

The elastic–plastic methods are currently adopted in modern standard codes of design to estimate the ultimate resistance of some steel structures, since they allow the beneficial effects of yielding in the redistribution of stresses to be taken into account. The analysis of the limit carrying capacity of a cross-section under biaxial bending is more simple than the analysis of its behaviour along the elastic–plastic range [7], and hence the earliest papers were restricted to that problem [33].

The research works carried out with this purpose have been based on analytical studies [14,17], experimental investigations [11,26,30], and numerical models [10,19,20]. A large number of these studies took into account other aspects than the elastic or plastic carrying capacity of the cross-sections, such as the possible occurrence of local or overall buckling phenomena of the structural elements in biaxial bending [26,31,32].

Although the results of some numerical models evidence a very good agreement with test results, their practical use for design purposes is limited, since most of them are currently not available and the labour required for the numerical calculation is quite extensive [20]. Therefore, their applications usually remain

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within the limits of research studies, and the designers often rely on simple interaction equations, between the cross-section internal forces, which may be found in bridge and building specifications such as [1–5,15], for instance.

The interaction criteria between the cross-section internal forces at its plastic limit state depend on the cross-section shape. Consequently, specific analytical expressions are required for each type of cross-sections. However, these analytical expressions are not currently available for some cross-section shapes, or they are defined by means of simplified equations, which do not take into account all the possible scenarios of loading, depending on the combinations of internal forces and relevant geometrical parameters.

On the other hand, the existing accurate methods are frequently complex, and difficult to apply in practice. This is often the case when biaxial bending of a cross-section is involved.

The design interaction formulae used to check the safety of members and cross-sections subjected to biaxial bending and axial force are usually the result of previous research studies, which are in the origin of those formulae or were dedicated to their discussion and validation. One of these interaction criteria, indicated in Eq. (1), was proposed by Bresler [12] and it has been adopted as the basis of the most common design criteria stated in the structural codes, for the verification of different types of cross-sections (solid and hollow rectangular sections, or H and I-shapes for instance) made of different materials, such as steel, aluminium, reinforced concrete, composite concrete and steel, etc:

$$\left(\frac{M_{n,y}}{M_{o,y}}\right)^{\alpha_1}+\left(\frac{M_{n,z}}{M_{o,z}}\right)^{\alpha_2}=1.0 \tag{1}$$

where $M_{n,y}$ and $M_{n,z}$ are the bending moment components, about the cross-section main axes of inertia, associated to an axial load N, and $M_{o,y}$ and $M_{o,z}$ represent the cross-section resistance capacities in simple bending under the axial load N, when $M_{n,z}=0$ or $M_{n,y}=0$ respectively. Many solutions have been suggested for the evaluation of the α_1 and α_2 coefficients or for alterations to Eq. (1) [23], in order to adjust it to the ultimate resistance capacity of different cross-section shapes and materials [15].

Rubin [28] has proposed new interaction criteria between the bending moment, the shear force and the axial force for simple symmetrical box and I-sections, when bent about their strong axis, and for double-symmetric I-sections bent about their weak axis. These equations are in the basis of the specifications from the Eurocode 3 [2,3] and from the German Steel Code DIN 18800 [4,5], for specific section types such as I-sections, circular tubes, rectangular hollow sections and solid rectangles and plates [25].

Yet, even if these equations give a good estimation of the cross-section resistance for a large number of practical situations, some research works have pointed out its limitations and have presented alternative solutions, namely under the form of design tables [17].

This work presents new interaction criteria for the analysis of rectangular hollow sections subjected to a combination of an axial force and biaxial bending moments, at the elastic or plastic limit states (as long as buckling phenomena are not involved). They have been obtained by means of an exact integration (within the frame of the hypotheses adopted in this study) of the cross-section axial stress field and they are valid for any current cross-section proportions (as long as buckling phenomena are not involved) and any position of the cross-section neutral axis. Shear stresses, due to bending or torsion for instance, are supposed to be very small and they are not taken into account.

Written in a non-dimensional form, these criteria are independent from the cross-section dimensions and steel strength, and from the Unit System used in the analysis [6].

The main advantages of these interaction criteria lie on their accuracy and on the direct relationship established between the cross-section internal forces, the cross-section geometric parameters and the position of the cross-section neutral axis. These advantages will be put in evidence by means of some application examples.

2. Basic principles of the analytical criteria

2.1. Assumptions

Fig. 1 presents two different configurations of a rectangular hollow cross-section under biaxial bending.

The v-axis is assumed to be the bending axis. Its direction is defined by the cross-section linear segment where the stresses due to biaxial bending, without axial loading, are equal to zero.

The cross-section b dimension is chosen parallel to the y-axis and the h dimension is parallel to the z-axis (Fig. 1). The values of M_y and M_z are supposed to be always positive; therefore, the inclination angle α of the bending axis v regarding the y-axis is within the limits $0 \leq \alpha < \pi/2$.

In the case of uniaxial bending about a main axis, we have $M_w=0$, $M_z=0$, $M_y=M_v$ and $\alpha=0$. If the bending axis is the strong axis, $b \leq h$; if the bending axis is the weak axis, $h \leq b$ (Fig. 1).

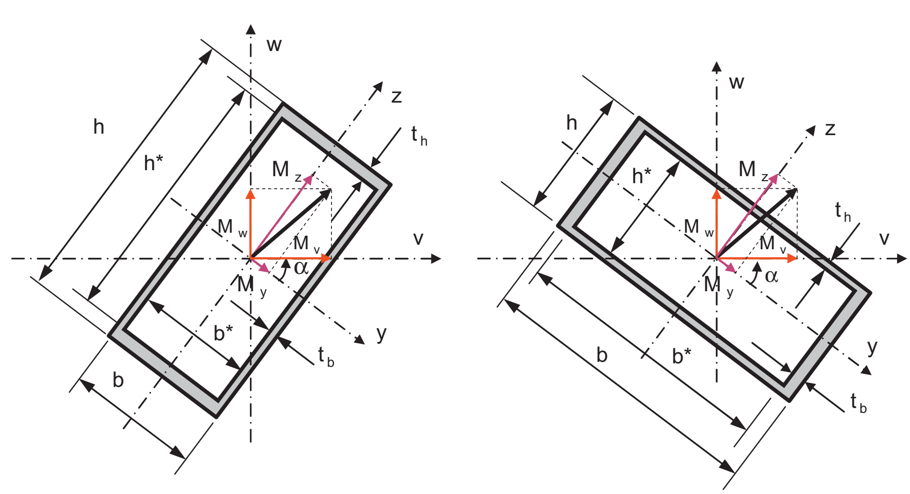

Fig. 1. Symbols and reference axes.