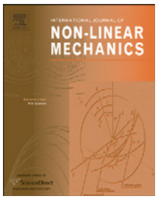
International Journal of Non-Linear Mechanics 47 (2012) 1055–1064

Contents lists available at SciVerse ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm



Non-linear analysis of functionally graded fiber reinforced composite laminated plates, Part II: Numerical results

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ARTICLE INFO

Article history:
Received 25 April 2011
Received in revised form
29 November 2011
Accepted 7 March 2012
Available online 21 March 2012

Keywords:
Functionally graded laminates
Hygrothermal effect
Temperature-dependent properties
Elastic foundation

ABSTRACT

In this Part, the extensive parametric studies performed are reported and numerical results are presented for the non-linear vibration, non-linear bending and compressive postbuckling of uniformly distributed and functionally graded fiber reinforced unsymmetric cross-ply and/or antisymmetric angle-ply laminated plates resting on Pasternak elastic foundations under different hygrothermal environmental conditions. The numerical results show that the functionally graded fiber reinforcement has a significant effect on the postbuckling response and load-bending moment curves of plate bending, whereas this effect is less pronounced on the load-deflection curves of plate bending and the linear and non-linear frequencies of the same plate.

1. Introduction

The solution methodology is described with sufficient detail in Part I. Results are presented herein for non-linear free vibration, non-linear bending and postbuckling of $(0/90)_S$ symmetric cross-ply, $(0/90)_{2T}$ unsymmetric cross-ply and $(45/-45)_{2T}$ antisymmetric angle-ply laminated plates resting on an elastic foundation in hygrothermal environments. The plate geometric parameters $a/b=1$, $b/h=10$, the thickness of each ply is identical and the total thickness of the plate $h=5$ mm. Four types of functionally graded fiber reinforced composite (FG-FRC) laminated plates are configurated. For Type V, the fiber volume fractions are assumed to have graded distribution [0.75/0.65/0.55/0.45] for four plies, referred to as FG-V. For Type Λ, the distribution of fiber reinforcements is inversed, i.e. [0.45/0.55/0.65/0.75], referred to as FG-Λ. For Type X_1, a mid-plane symmetric graded distribution of fiber reinforcements is achieved, i.e. [0.75/0.45/0.45/0.75], and for type X_2 the fiber volume fractions are assumed to have [0.45/0.75/0.75/0.45], referred to as FG-X_1 and FG-X_2, respectively. A uniformly distributed fiber reinforced composite (UD-FRC) laminated plate with the same thickness is also considered as a comparator for which the fiber volume fraction of each ply is identical and $V_f=0.6$. In such a way, the two cases of UD- and FG-FRC laminated plates will have the same value of total fraction of fiber.

For all cases discussed below, graphite/epoxy composites are selected. Unlike in [1–4], the material properties of fibers are assumed to be anisotropic and are taken to be [5] $E_{11}^f=233.05\,\text{GPa}$, $E_{22}^f=23.1\,\text{GPa}$, $G_{12}^f=8.96\,\text{GPa}$, $\nu^f=0.2$, $\alpha_{11}^f=-0.54\times10^{-6}/{}^\circ\text{C}$, $\alpha_{22}^f=10.08\times10^{-6}/{}^\circ\text{C}$, $\rho^f=1750\,\text{kg/m}^3$. The material properties of matrix are assumed to be $c_{fm}=0$, $\nu^m=0.34$, $\alpha^m=45.0\times10^{-6}/{}^\circ\text{C}$, $\rho^m=1200\,\text{kg/m}^3$, $\beta^m=2.68\times10^{-3}$/wt percent H_2O, and $E^m=(3.51-0.003T-0.142C)$ GPa, in which $T=T_0+\Delta T$ and $T_0=25$ C (room temperature), and $C=C_0+\Delta C$ and $C_0=0$ wt percent H_2O.

Two foundation models are considered. The stiffnesses are $(k_1, k_2)=(100, 10)$ for the Pasternak elastic foundation, $(k_1, k_2)=(100, 0)$ for the Winkler elastic foundation and $(k_1, k_2)=(0, 0)$ for the plate without any elastic foundation. The in-plane boundary condition is assumed to be immovable (case 2) except for the Table 7 in Section 2 and Fig. 7 in Section 3, whereas in Section 4 the in-plane boundary condition is assumed to be movable (case 1).

2. Non-linear vibration of FG-FRC laminated plates

Before generating extensive results, a few check cases are considered in order to test the derived solutions.

As first example, the first four dimensionless natural frequencies of a $(0/90)_S$ symmetric cross-ply laminated plate at $C=0.1\%$ and $T=325$ K are calculated and compared in Table 1 with Ritz method results of Whitney and Ashton [6], finite element method (FEM) results of Ram and Sinha [7] and Parhi et al. [8], and perturbation solutions of Huang et al. [4]. The geometric parameters and material properties adopted are: $a/b=1$, $b/h=100$,

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http://dx.doi.org/10.1016/j.ijnonlinmec.2012.03.003

$E_{11}=130$ GPa, $E_{22}=9.5$ GPa, $G_{12}=G_{13}=6.0$ GPa, $G_{23}=0.5G_{12}$, $\nu_{12}=0.3$, $\alpha_{11}=-0.3\times10^{-6}$/K, $\alpha_{22}=28.1\times10^{-6}$/K. The dimensionless frequencies are defined by $\overline{\omega}=\Omega(a^2/h)\sqrt{\rho/E_{22}}$.

As a second example, the first four dimensionless natural frequencies for a (0/0/0/90/0) unsymmetric cross-ply laminated square plate with each ply having different thickness and material properties are calculated and compared in Table 2 with FEM results of Kulkarni and Kapuria [9] based on a higher order shear deformation plate theory (HSDPT). The thicknesses of each ply are $[0.1h/0.25h/0.15h/0.2h/0.3h]$ and the material properties are $E_{11}=E_{22}=6.9$ GPa, $G_{12}=G_{13}=G_{23}=1.38$ GPa, for the first ply; $E_{11}=224.25$ GPa, $E_{22}=6.9$ GPa, $G_{12}=G_{13}=56.58$ GPa, $G_{23}=1.38$ GPa for the second ply; and $E_{11}=172.5$ GPa, $E_{22}=6.9$ GPa, $G_{12}=G_{13}=3.45$ GPa, $G_{23}=1.38$ GPa for the other three plies; and for all these plies $\nu_{12}=0.25$, $\rho=1578$ kg/m^3.

Table 1
Comparisons of natural frequency parameters $\overline{\omega}=\Omega_L(a^2/h)\sqrt{\rho/E_{22}}$ for $(0/90)_S$ laminated square thin plates in hygrothermal environments.

Source	$\overline{\omega}_{11}$	$\overline{\omega}_{12}$	$\overline{\omega}_{21}$	$\overline{\omega}_{22}$
$C=0.1\%$				
Whitney and Ashton [6]	9.411	19.911	39.528	45.815
Ram and Sinha [7]	9.429	20.679	40.068	46.752
Parhi et al. [8]	9.393	19.887	39.345	–
Huang et al. [4]	9.389	19.866	39.265	45.518
Present	9.389	19.866	39.265	45.520
$T=325$ K				
Whitney and Ashton [6]	8.068	18.378	38.778	44.778
Ram and Sinha [7]	8.088	19.196	39.324	45.431
Parhi et al. [8]	8.046	18.350	38.590	–
Huang et al. [4]	8.043	18.140	38.364	44.686
Present	8.042	18.329	38.511	44.476

Table 2
Comparisons of natural frequency parameters $\overline{\omega}=\Omega_L(a^2/h)\sqrt{\rho/E_{22}}$ for a (0/0/0/90/0) laminated square plate with each ply having different thickness and material properties.

b/h	Source	$\overline{\omega}_1$	$\overline{\omega}_2$	$\overline{\omega}_3$	$\overline{\omega}_4$
5	Kulkarni and Kapuria [9]	12.5024	17.0884	23.1636	32.4004
	Present	12.5336	17.1726	23.2941	32.5331
10	Kulkarni and Kapuria [9]	14.7135	21.9033	32.1707	44.4610
	Present	14.7391	21.9950	32.3403	44.6096
20	Kulkarni and Kapuria [9]	15.5240	24.1336	37.4023	50.7163
	Present	15.5435	24.2037	37.5012	50.8190

In addition, the non-linear to linear frequency ratios ω_{NL}/ω_L for cross-ply laminated plates are calculated and compared in Table 3 with the direct integration method results of Singh et al. [10], and the homotopy analysis method results of Pirbodaghi et al. [11]. The geometric parameters and material properties adopted are: $b=0.254$ m, $E_{11}=206.84$ GPa, $E_{22}=5.171$ GPa, $G_{12}=G_{13}=G_{23}=2.855$ GPa, $\nu_{12}=0.254$, $\rho=2564.8$ kg/m^3.

These three comparisons show that the results from the present method are in good agreement with the existing results, thus verifying the reliability and accuracy of the present method. Note that in all these three examples the material properties are assumed to be independent of temperature.

Table 4 presents the first five dimensionless natural frequencies of $(0/90)_S$, $(0/90)_{2T}$ and $(45/-45)_{2T}$ laminated plates with different types of FG distribution of fiber reinforcements at $\Delta T=\Delta C=0$. The results for the same plate with UD distribution of fiber reinforcements are also listed for direct comparison. The dimensionless natural frequency is defined by $\tilde{\Omega}=\Omega(a^2/h)\sqrt{\rho_0/E_0}$, where ρ_0 and E_0 are the reference values of ρ^m and E^m at $\Delta T=\Delta C=0$. It can be seen that the functionally graded distribution of FG-V and FG-Λ has a very small effect on the natural frequencies of $(0/90)_S$ and $(0/90)_{2T}$ plates. It is found that the $(45/-45)_{2T}$ plate of FG-X$_1$ type, while the $(0/90)_S$ and $(0/90)_{2T}$ plates of FG-X$_2$ type have a lower natural frequency than the same plate of UD type.

Tables 5 and 6 show, respectively, the effect of foundation stiffness along with the hygrothermal effect on the fundamental

Table 4
Comparisons of natural frequencies $\tilde{\Omega}=\Omega(a^2/h)\sqrt{\rho_0/E_0}$ for hybrid laminated plates ($a/b=1$, $b/h=10$).

Lay-up		$\tilde{\Omega}_{11}$	$\tilde{\Omega}_{12}$	$\tilde{\Omega}_{21}$	$\tilde{\Omega}_{22}$	$\tilde{\Omega}_{13}$
$(0/90)_S$	UD	13.639	25.299	31.929	38.659	43.102
	FG-V	13.589	25.424	32.089	38.922	43.426
	FG-Λ	13.589	25.424	32.089	38.922	43.426
	FG-X$_1$	14.165	24.884	32.642	38.834	41.581
	FG-X$_2$	13.301	26.521	32.273	39.916	46.438
$(0/90)_{2T}$	UD	13.770	30.524	30.524	41.154	50.760
	FG-V	13.728	30.213	30.929	41.273	50.502
	FG-Λ	13.727	30.921	30.213	41.270	51.521
	FG-X$_1$	13.776	30.096	30.096	40.530	49.691
	FG-X$_2$	13.643	31.386	31.386	42.554	53.505
$(45/-45)_{2T}$	UD	16.806	31.717	31.717	45.810	49.465
	FG-X$_1$	16.601	31.179	31.179	44.740	48.517
	FG-X$_2$	16.954	32.804	32.804	48.273	51.876

Table 3
Comparison of non-linear to linear frequency ratios ω_{NL}/ω_L for cross-ply laminated plates ($b=0.254$ m).

a/b	b/h	Source	$\overline{W}_{\max}/h$					
			0.25	0.5	0.75	1.0	1.5	2.0
(0/90/90/0)								
1.0	40	Singh et al. [10]	1.0535	1.2038	1.4172	1.6691	2.2355	2.8439
		Pirbodaghi et al. [11]	1.0501	1.1879	1.3874	1.6245	2.1659	2.7486
		Present	1.0503	1.1885	1.3886	1.6279	2.1709	2.7569
2.0	20	Singh et al. [10]	1.1327	1.4674	1.8946	2.3652	3.3634	4.3949
		Pirbodaghi et al. [11]	1.1276	1.4434	1.8541	2.3092	3.2783	4.2811
		Present	1.1286	1.4475	1.8612	2.3196	3.2949	4.3038
(0/90/0/90)								
1.0	40	Singh et al. [10]	1.0634	1.2388	1.4832	1.7679	2.4000	3.0729
		Pirbodaghi et al. [11]	1.0514	1.1924	1.3961	1.6392	2.1899	2.7835
		Present	1.0590	1.2189	1.4467	1.7155	2.3178	2.9618
2.0	20	Singh et al. [10]	1.0653	1.2454	1.4956	1.7863	2.4303	3.1148
		Pirbodaghi et al. [11]	1.0625	1.2112	1.4719	1.7608	2.3712	3.0031
		Present	1.0642	1.2369	1.4807	1.7664	2.4021	3.0790