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Flow due to non-coaxial rotation of a porous disk and a second grade fluid rotating at infinity

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A B S T R A C T

An exact solution for the three-dimensional flow due to non-coaxial rotation of a porous disk and a second grade fluid at infinity is obtained. It is shown that for uniform suction or uniform blowing at the disk, an asymptotic profile exists for the velocity distribution. The velocity depends on two parameters: one of them is the suction parameter or blowing parameter and the other is the visco-elastic parameter. Furthermore, it is found that when the value of the visco-elastic parameter is fixed, the velocity decreases with an increase in the value of the suction parameter and when the value of the suction parameter is fixed, the velocity increases with an increase in the value of the visco-elastic parameter.

1. Introduction

An exact solution for the flow of an incompressible second grade fluid due to non-coaxial rotation of a porous disk in an infinite region is obtained. Obtaining an exact solution for a flow of a second grade fluid is very important, because the governing equation of a second grade fluid has two non-linear terms: one of them is due to the inertia term and the other is due to visco-elastic term. Exact solutions provide a standard for checking the accuracies of many approximate methods such as numerical or empirical. The accuracy of the results can be established by a comparison with an exact solution. The flow over boundaries of porous materials has many applications in practice such as boundary layer control.

The problem considered in this paper is an extension of the flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity to that of a second grade fluid. It is well known that the governing equation of a second grade fluid is a third order partial differential equation. The no-slip condition provides two conditions, then, one needs an additional condition. It is shown that in the problem considered in this paper due to the extension of the flow region to infinity, no-slip condition is sufficient.

The steady flow in an infinite region due to non-coaxial rotation of a porous disk has been investigated by many authors. Flow due to eccentrically rotating porous disk and a fluid at infinity was studied in [1], assuming that the porous disk and the fluid at infinity are rotating with the same angular velocity. Flow due to eccentric rotation of a porous disk and a fluid at infinity, but with different angular velocity, has been investigated in [2]. The MHD flow and heat transfer due to eccentric rotation of a porous disk and a fluid at infinity was studied in [3]. An unsteady flow due to eccentrically rotating porous disk and a fluid at infinity in the case of the porous disk executing non-torsional oscillations in its own plane was studied in [4]. The MHD flow due to non-coaxial rotation of a porous disk and a fourth grade fluid at infinity has been investigated in [5]. The flow induced by non-coaxial rotation of a porous disk executing non-torsional oscillations and a fluid of second grade at infinity was studied in [6].

In this paper, the three-dimensional flow of a second grade fluid is studied. It is shown that for uniform suction or uniform blowing at the disk, an asymptotic profile exists for the velocity distribution. The velocity depends on two parameters; one of them is the suction or blowing parameter and the other is the visco-elastic parameter. For the large values of the suction parameter, the velocity changes appreciably near the porous plate. However, the velocity increases with increase in the value of the visco-elastic parameter.

2. Basic equations

The equation of motion for a fluid in the absence of the body forces is

$$\rho \frac{D\boldsymbol{u}}{Dt} = \nabla \cdot \boldsymbol{\sigma} \tag{1}$$

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where ρ is the density of the fluid, $\boldsymbol{u}$ is the velocity, $\boldsymbol{\sigma}$ is the stress tensor and D/Dt represents the material derivative. The continuity equation for the velocity is

$$\nabla \cdot \boldsymbol{u} = 0 \tag{2}$$

Eqs. (1) and (2) can be applied to all types of incompressible Newtonian and non-Newtonian fluids. The stress depends on the local properties of the fluids. The relation between the stress and the local properties of the fluid is called the constitutive equation. The constitutive equation for an incompressible second grade fluid is in the following form [7]:

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \mu\boldsymbol{A}_1 + \alpha_1\boldsymbol{A}_2 + \alpha_2\boldsymbol{A}_1^2 \tag{3}$$

where μ, α_1 and α_2 are the material constants and A_n represents the Rivlin–Ericksen tensor defined in [8] as

$$\boldsymbol{A}_0 = \boldsymbol{I}, \quad \boldsymbol{A}_1 = \nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T$$
$$\boldsymbol{A}_{n+1} = \left(\frac{\partial}{\partial t} + \boldsymbol{u}\cdot\nabla\right)\boldsymbol{A}_n + \boldsymbol{A}_n\cdot\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\cdot\boldsymbol{A}_n \tag{4}$$

where t is the time, p is the pressure and $\boldsymbol{I}$ is the identity tensor. The Clausius–Duhem inequality and the condition that Helmholtz free energy is minimum in equilibrium provide the following restrictions [9,10]:

$$\mu \ge 0, \quad \alpha_1 + \alpha_2 = 0, \quad \alpha_1 \ge 0 \tag{5}$$

A comprehensive discussion on the restrictions for μ, α_1 and α_2 can be found in [11].

When Eq. (3) is substituted into Eq. (1), one obtains the following equation [12]:

$$\frac{\partial\boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{1}{\rho}\nabla p + v\nabla^2\boldsymbol{u} + \beta(\nabla^2\boldsymbol{\omega})\boldsymbol{u} \tag{6}$$

where condition (5) is used and $\omega = \nabla\times\boldsymbol{u}$.

The physical model and coordinate system are denoted in Fig. 1. A Cartesian coordinate system in which z-axis is normal to the porous disk and the plane of the disk is $z=0$ is introduced. The axis of rotation of the disk and that of fluid at infinity are assumed to be in the plane $x=0$, and the distance between the axes is l. The disk and the fluid at infinity are rotating about the axes with the same angular velocity Ω. The boundary conditions are

$$u = -\Omega y, \quad \upsilon = \Omega x, \quad w = -w_0 \text{ at } z=0$$
$$u = -\Omega y + \Omega l, \quad \upsilon = \Omega x, \quad w = -w_0 \text{ at infinity} \tag{7}$$

where u, υ and w are the Cartesian components of velocity. The boundary conditions given by (7) suggests a velocity field in the following form:

$$u = -\Omega y + f(z), \quad \upsilon = \Omega x + g(z), \quad w = -w_0 \tag{8}$$

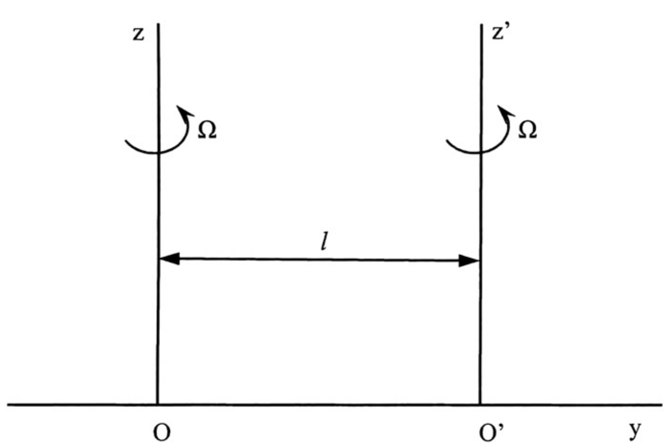

Fig. 1. Flow geometry and coordinate systems.

The velocity field can be considered as the summation of a helical motion $(-\Omega y, \Omega x, -w_0)$ and a translational motion $(f(z), g(z), 0)$. The boundary conditions given by (7) require

$$f(0)=0, \quad g(0)=0, \quad f(\infty)=\Omega l, \quad g(\infty)=0 \tag{9}$$

Substituting the velocity components given by (8) into (6), one finds

$$-(\Omega^2 x + \Omega g + w_0 f') = -\frac{1}{\rho}\frac{\partial p}{\partial x} + vf'' - \beta w_0 f''' \tag{10}$$

$$-(\Omega^2 y - \Omega f + w_0 g') = -\frac{1}{\rho}\frac{\partial p}{\partial y} + vg'' - \beta w_0 g''' \tag{11}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \left[(\Omega x + g)g''' + (-\Omega y + f)f'''\right] \tag{12}$$

Differentiation of $\partial p/\partial x$ and $\partial p/\partial y$ with respect to z and $\partial p/\partial z$ with respect to x and y gives

$$-\beta w_0 f''' + vf'' + \beta\Omega g'' + w_0 f' + \Omega g = C_1 \tag{13}$$

$$-\beta w_0 g''' + vg'' - \beta\Omega f'' + w_0 g' - \Omega f = C_2 \tag{14}$$

Eqs. (13) and (14) are general and can be applied to the case of non-coaxial rotation of two disks. The condition at infinity provides $C_1=0$ and $C_2=-\Omega^2 l$.

3. Solution of the problem

Some special cases are considered. When w_0 is zero, Eqs. (13) and (14) reduce to

$$vf'' + \beta\Omega g'' + \Omega g = 0, \quad vg'' - \beta\Omega f'' - \Omega f = -\Omega^2 l \tag{15}$$

The solutions of Eq. (15) are

$$\frac{f}{\Omega l} = 1 - e^{-a\xi}\cos b\xi, \quad \frac{g}{\Omega l} = e^{-a\xi}\sin b\xi \tag{16}$$

where

$$\xi = (\Omega/2v)^{1/2}z, \quad a = \{[(1+\varepsilon^2)^{1/2} - \varepsilon]/(1+\varepsilon^2)\}^{1/2}$$
$$b = \{[(1+\varepsilon^2)^{1/2} + \varepsilon]/(1+\varepsilon^2)\}^{1/2}, \quad \varepsilon = \beta\Omega/v$$

When $\beta=0$, Eqs. (13) and (14) reduce to

$$vf'' + w_0 f' + \Omega g = 0, \quad vg'' + w_0 g' - \Omega f = -\Omega^2 l \tag{17}$$

The solutions have been given in [1] in the following form:

$$\frac{f}{\Omega l} = 1 - e^{-c\xi}\cos d\xi, \quad \frac{g}{\Omega l} = e^{-c\xi}\sin d\xi \tag{18}$$

where

$$c = \sqrt{2}s + \sqrt{(s^4+1)^{1/2} + s^2}, \quad d = \sqrt{(s^4+1)^{1/2} - s^2}, \quad s = w_0/2(\Omega v)^{1/2} \tag{19}$$

The blowing case solution is given by

$$\frac{f}{\Omega l} = 1 - e^{-\gamma\xi}\cos\delta\xi, \quad \frac{g}{\Omega l} = e^{-\gamma\xi}\sin\delta\xi \tag{20}$$

where

$$\gamma = \sqrt{(\lambda^4+1)^{1/2} + \lambda^2} - \sqrt{2}\lambda, \quad \delta = \sqrt{(\lambda^4+1)^{1/2} - \lambda^2}, \quad \lambda = -s \tag{21}$$

For $w_0=0$ and $\beta=0$, Eqs. (13) and (14) reduce to

$$vf'' + \Omega g = 0, \quad vg'' - \Omega f = -\Omega^2 l \tag{22}$$

The solutions are

$$f = 1 - e^{-\xi}\cos\xi, \quad g = e^{-\xi}\sin\xi \tag{23}$$