International Journal of Non-Linear Mechanics 46 (2011) 407–414

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Suppression of pull-in in a microstructure actuated by mechanical shocks and electrostatic forces

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ARTICLE INFO

Article history:
Received 2 September 2009
Received in revised form
28 October 2010
Accepted 29 October 2010

Keywords:
Pull-in
Capacitive MEMS
High-frequency voltage
Shock
Control

ABSTRACT

This work investigates the effect of a high-frequency voltage (HFV) on the pull-in instability in a microstructure actuated by mechanical shocks and electrostatic forces. The microstructure is modelled as a single-degree-of-freedom mass-spring-damper system. The method of direct partition of motion is used to split the fast and slow dynamics. Analysis of steady-state solutions of the slow dynamic allows the investigation of the influence of the HFV on the pull-in. The results show that adding HFV rigidifies the system, creates new stable equilibria and suppresses the pull-in instability for adequate high-frequency voltages. To illustrate the applicability of the result, a specific capacitive microelectromechanical system consisting of a clamped–clamped microbeam is considered.

1. Introduction

Analysis of vibrational behavior of Microelectromechanical systems (MEMS) is an active topic of research with applications in many engineering fields such as communications, automotive, robotics and others. One of the most critical issues in the design of MEMS is their reliability and survivability under mechanical shocks and electrical loads. In the case of capacitive MEMS devices the pull-in [1–3] constitutes one of the main roots to the device failure. Pull-in is a structural instability phenomenon resulting from the interaction between elastic and electrostatic forces in MEMS devices. This instability results from the unbalance between the electric actuation and the mechanical restoring force leading a movable electrode to hit a stationary electrode causing stiction and short circuit problems and hence the failure in the device's function [4]. Several works [5,6] investigated the static pull-in phenomenon and performed techniques to predict its occurrence by determining the largest DC voltage for which the system operates in a stable behavior. The dynamic pull-in was studied under various loads, such that step voltage [7], AC harmonic voltage [8,9] and mechanical shock load [10,11]. It was shown that the dynamic actuation reduces drastically the static pull-in threshold. Nayfeh and co-workers [8,9] studied the dynamic pull-in of MEMS resonators actuated by a resonant AC voltage. They found three distinct mechanisms leading to the dynamic pull-in instability. The first mechanism is the cyclic-fold or symmetry instabilities, the second mechanism depends on the system transient dynamics and the number of coexisting attractors and the third one is characterized by the sensibility to initial conditions due to the existence of homoclinic tangles. Moreover, Younis and co-workers [10,11] showed that the combination of a shock load and an electrostatic actuation makes the instability threshold much lower than the threshold predicted considering the effect of the shock alone or the electrostatic actuation alone. They also studied the effects of the shape of the shock pulse and its duration on the pull-in threshold. Recently, Ibrahim and Younis [12] presented a theoretical and experimental investigation of the response of electrostatically actuated parallel-plate resonators subjected to mechanical shocks. They concluded that a resonator may experience early dynamic pull-in instability depending on the shock duration.

Keeping a MEMS device operating in a stable attracting regime away from the pull-in instability limit presents a major interest from design, fabrication process and commercialization point of view. This challenge has motivated researchers developing strategies to avoid the pull-in, and hence, increase the range of movable electrode. For example, Castañer et al. [13] used an interesting technique based on charge control, instead of voltage control. This method allows extending the travel range, but it is limited by the charge pull-in [14]. Lenci and Rega [15] used a control method based on adding superharmonics to a reference harmonic excitation and showed the possibility of shifting the dynamic pull-in towards high excitation amplitudes. Lakrad and Belhaq [16] showed that applying an appropriate high-frequency harmonic voltage can delay the static pull-in.

In the present paper, a HFV is used to suppress the pull-in instability induced by the combined effect of electrostatic and shock forces. The proposed method is applied to a simplified mass-spring-damper

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doi:10.1016/j.ijnonlinmec.2010.10.006

system modelling the dynamic of a capacitive MEMS. It is worth noting that the problem of studying the effects of high-frequency excitations on the dynamic of mechanical systems has been examined during the last decade by a number of authors; see for instance [17] and references therein. Attention was focused on the effect of high-frequency excitations on the natural frequencies [18,19], symmetry breaking [20], limit cycles [21], hysteresis [22,23] and pull-in instability [16].

The rest of the paper is organized as follows. The equation of motion modelling the dynamic of the MEMS device is presented as a mass-spring-damping oscillator in Section 2. Then, the method of direct partition of motion is performed on the oscillator over the fast dynamic and the main equation governing the slow dynamic of the MEMS device is derived. In Section 3, we expose the main analytical and numerical results of our study, while in Section 4 an application to a real capacitive MEMS is considered. Section 5 concludes the work.

2. Formulation of the problem and slow dynamic

Consider the following non-dimensional equation

$$X''+2\xi X'+X=\frac{\alpha}{(1-X)^2}+\frac{\beta\cos(\Omega\tau)}{(1-X)^2}+f_0 g(\tau) \tag{1}$$

representing a single-degree-of-freedom model of a capacitive MEMS device employing a DC and AC voltages as actuator and subjected to a mechanical shock. The primes denote the derivatives with respect to the non-dimensional time τ, X is the normalized displacement with respect to the initial gap of the movable mass and ξ is the damping coefficient. The amplitude and the pulse shape of the shock are denoted by f_0 and $g(\tau)$, respectively. Note that $X=1$ corresponds to the pull-in and the left hand side of Eq. (1) is considered as a linear mechanical oscillator. However, non-linearities can arise in the mechanical subsystem through non-linear mechanical stiffness. The choice of considering a linear mechanical model can be justified by the fact that the thickness of the movable electrode is greater than the initial gap. The first term in the right hand side of Eq. (1) represents the effect of the DC voltage, the second term is related to the AC actuation and the last one describes the effect of the shock load. The parameters α and β are first treated as independent entities in the analysis, even though they are related in real capacitive MEMS. This does not impact our results as shown in Section 4. Note that in [16] the authors shown that it is possible, in this case, to prevent the electrostatically induced pull-in instability for a range of values of the amplitude and the frequency of the high-frequency AC. The high-frequency Ω is normalized with respect to the natural frequency and is taken very large with respect to unity. In this paper, we consider that the natural period T_p of the microstructure is very small compared to the duration T of the shock. Consequently, the shock force is experienced as a quasi-static force that stays for some time and is then removed [10].

Eq. (1) contains a slow dynamic which describes the main motion at time-scale of the microstructure natural vibration and a fast dynamic at time-scale of the high-frequency voltage. To obtain the main equation governing the slow dynamic of the device, we implement the method of direct partition of motion [17]. Two different time-scales are introduced: a fast time $T_0=\eta^{-1}\tau$ and a slow time $T_1=\tau$ and the displacement of the mass $X(\tau)$ is split up into a slow part $Z(T_1)$ and a fast part $\phi(T_0,T_1)$ as follows:

$$X(\tau)=Z(T_1)+\phi(T_0,T_1)=Z(T_1)+\eta^2\tilde{\phi}(T_0,T_1) \tag{2}$$

Here the positive parameter η is introduced to measure the smallness of other parameters ($0<\eta\ll 1$). The slow part $Z(T_1)$ takes into account the transient motion composed of the natural damped motion of the microstructure and the response to the shock force. The high-frequency is taken as $\Omega=\eta^{-1}$. The fast motion and its derivatives are assumed to be 2π-periodic functions of the fast time T_0 with zero mean value with respect to it. Thus, $\langle X(\tau)\rangle=Z(T_1)$ where $\langle . \rangle=(1/2\pi)\int_0^{2\pi}(.)dT_0$ defines the fast time-averaging operator. Introducing $D_m^n=\partial^n/\partial T_m^n$ yields

$$\frac{d}{d\tau}=\eta^{-1}D_0+D_1+O(\eta) \tag{3}$$

$$\frac{d^2}{d\tau^2}=\eta^{-2}D_0^2+\eta^{-1}2D_0D_1+D_1^2+O(\eta) \tag{4}$$

Substituting (3) and (4) into (1) leads to the following equation

$$\begin{aligned}(D_0^2\tilde{\phi})&+2\eta(D_0D_1\tilde{\phi})+\eta^2(D_1^2\tilde{\phi})+(D_1^2Z)\\&+2\tilde{\xi}[(D_0\tilde{\phi})+\eta(D_1\tilde{\phi})+(D_1Z)]+Z+\eta^2\tilde{\phi}\\&=\frac{\alpha}{(1-Z-\eta^2\tilde{\phi})^2}+\frac{\beta\cos(T_0)}{(1-Z-\eta^2\tilde{\phi})^2}+f_0g(T_1)\end{aligned} \tag{5}$$

In what follows we set $\beta=O(1)$ and $\xi=\eta\tilde{\xi}$. All the parameters with tildes are of order $O(1)$.

The dominant terms depending on T_0 up to the order $O(1)$ in (5) are

$$(D_0^2\tilde{\phi})=\frac{\beta}{(1-Z-\eta^2\tilde{\phi})^2}\cos(T_0) \tag{6}$$

Thus, up to this leading order, the fast motion is given by

$$\tilde{\phi}(T_0,T_1)=-\frac{\beta}{(1-Z)^2}\cos(T_0)+O(\eta) \tag{7}$$

Now averaging Eq. (5) over a period of the fast time scale T_0 leads to the following equation

$$\begin{aligned}(D_1^2Z)&+2\xi(D_1Z)+Z\\&=\left\langle\frac{\alpha}{(1-Z-\eta^2\tilde{\phi})^2}+\frac{\beta\cos(T_0)}{(1-Z-\eta^2\tilde{\phi})^2}\right\rangle+f_0g(T_1)\end{aligned} \tag{8}$$

where

$$\left\langle\frac{\alpha}{(1-Z-\eta^2\tilde{\phi})^2}\right\rangle=\frac{\alpha}{(1-Z)^2}+O(\eta^3) \tag{9}$$

$$\left\langle\frac{\beta}{(1-Z-\eta^2\tilde{\phi})^2}\cos(T_0)\right\rangle=-\frac{\beta^2}{\Omega^2(1-Z)^5}+O(\eta^3) \tag{10}$$

The equation of the slow dynamics up to the order $O(\eta^3)$ can be written as

$$(D_1^2Z)+2\xi(D_1Z)+Z=\frac{\alpha}{(1-Z)^2}-\frac{\beta^2}{\Omega^2(1-Z)^5}+f_0g(T_1) \tag{11}$$

Eq. (11) governs the transient slow dynamics of the movable electrode and can be used to study the pull-in instability in the presence of a high-frequency harmonic actuation. The present work focuses attention only on steady-state equilibria. Through Eq. (11) one can conclude that the DC voltage is softening the structure while the HFV is hardening it. As a consequence, increasing β will increase the natural frequency of the structure and will damp quickly its free vibrations. It is worth noting that the zeros of the slow dynamic, Eq. (11), in the absence of the shock or after the end of its effect, are the periodic solutions of Eq. (1). These zeros are obtained by solving the following sixth order algebraic equation

$$Z(1-Z)^5=\alpha(1-Z)^3-\frac{\beta^2}{\Omega^2} \tag{12}$$

It is clear that in the absence of the HFV i.e., for $\beta=0$, the steady-state equilibria are obtained by solving $Z(1-Z)^2=\alpha$. The static pull-in in this case is obtained for the static tension $\alpha_p=\frac{4}{27}=0.148$ which corresponds to the steady state displacement $Z_p=\frac{1}{3}$.