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Unsteady MHD mixed convection flow over an impulsively stretched permeable vertical surface in a quiescent fluid

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A B S T R A C T

The unsteady mixed convection flow of an incompressible laminar electrically conducting fluid over an impulsively stretched permeable vertical surface in an unbounded quiescent fluid in the presence of a transverse magnetic field has been investigated. At the same time, the surface temperature is suddenly increased from the surrounding fluid temperature or a constant heat flux is suddenly imposed on the surface. The problem is formulated in such a way that for small time it is governed by Rayleigh type of equation and for large time by Crane type of equation. The non-linear coupled parabolic partial differential equations governing the unsteady mixed convection flow under boundary layer approximations have been solved analytically by using the homotopy analysis method as well as numerically by an implicit finite difference scheme. The local skin friction coefficient and the local Nusselt number are found to decrease rapidly with time in a small time interval and they tend to steady-state values for $t^* \geq 5$. They also increase with the buoyancy force and suction, but decrease with injection rate. The local skin friction coefficient increases with the magnetic field, but the local Nusselt number decreases. There is a smooth transition from the unsteady state to the steady state.

1. Introduction

The flow and heat transfer characteristics in the boundary layer induced by a continuous surface moving with uniform or non-uniform velocity in a quiescent fluid are important in several manufacturing processes in industry such as the extrusion of a plastic sheet, the cooling of a metallic plate in a cooling bath, wire drawing, hot rolling etc. Glass blowing, fibre production, crystal growing and paper production also involve the flow due to a stretching surface. This problem differs from the classical boundary layer flow over a stationary surface due to the entrainment of the fluid. A moving surface delays or prevents the separation of boundary layer from the wall by importing momentum into the boundary layer. The buoyancy force, suction or injection and the magnetic field significantly influence the skin friction and heat transfer. Therefore, the unsteady mixed convection flow over an impulsively stretched surface in the presence of a magnetic field and suction or injection is an important problem.

Sakiadis [1] was the first to study the flow induced by a surface moving with constant velocity in a quiescent fluid. The corresponding heat transfer problem was studied by Tsou et al. [2], Erickson et al. [3] and Griffin and Thorne [4]. Crane [5] considered the same problem as in [1], but assumed that the surface velocity varies linearly with the distance x ($U = ax, a > 0$) and obtained a closed form solution. Carragher and Crane [6] investigated the heat transfer problem on a linearly stretching impermeable isothermal surface and obtained an analytical solution. Stretched surfaces with different velocities and (or) temperature conditions at the surface were studied by Grubka and Bobba [7], Jeng et al. [8], Ali [9,10], Magyari and Keller [11], Magyari et al. [12], Vajravelu [13], Liao [14], Sparrow and Abraham [15], Vajravelu and Cannon [16] and Andersson and Aarseth [17].

In recent years, MHD problems have become more important industrially. Many metallurgical processes, such as drawing, annealing and tinning of copper wires, involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. By drawing such strips in an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and a final product of desired quality can be obtained. Another important application of hydromagnetics to metallurgy is the purification of molten metals from non-metallic inclusions by the application of a magnetic field. The effect of the magnetic field on the flow over a stretching surface with or without heat and mass transfer was studied by a number of research workers [18–30] who examined various aspects of this problem which include the effects of uniformly or non-uniformly stretched

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surface, suction or injection, heat generation or absorption, chemical reaction, variable viscosity, thermal stratification, uniform free stream velocity, stagnation flow and rotating fluid.

It is known that the buoyancy force can produce significant changes in the velocity and temperature distribution and, hence in the heat transfer rate from the surface. The effect of buoyancy force over continuous moving surfaces through an otherwise quiescent fluid was investigated by Chen and Strobel [31] and Fan et al. [32] for horizontal surfaces, by Chen [33], Ali and Al-Yousef [34,35] and Fan et al. [36] for vertical surfaces and by Moutsoglou and Chen [37], Strobel and Chen [38] and Chen [39] for vertical and inclined surfaces. In the presence of buoyancy forces, similarity solutions exist only for a certain power-law distribution of surface velocity and temperature. Chen and Armaly [40] reviewed mixed convection correlations for moving sheets and presented extensive correlations for horizontal, inclined and vertical moving surfaces and for both uniform wall temperature and uniform heat flux boundary conditions in buoyancy-assisting and buoyancy-opposing cases. In a recent review, Viskanta and Bergman [41] discussed several aspects related to moving surface problem. Al-Sanea [42,43] studied the effects of the buoyancy force with or without surface mass transfer on the flow and thermal fields close to and far downstream from the extrusion slit for different Prandtl number fluids and buoyancy parameter. The Navier–Stokes equations and the energy equation governing the flow and heat transfer were solved by using the finite volume method. These results were used to map out the entire forced, mixed and natural convection regimes.

The studies reported above considered the steady flow. However, the flow and heat transfer problem will become unsteady due to the impulsive change in the surface velocity or wall temperature (heat flux) or due to the time-dependent variations in them. The unsteady flow and heat transfer on a stretching surface is an important problem, since it is not always possible to maintain steady-state conditions. Pop and Na [44] investigated the unsteady flow over a stretching surface, where the surface is impulsively stretched at time t=0 with a velocity which varies linearly with the distance x along the surface. Elbashbeshy and Bazid [45,46] considered the heat transfer problem over an unsteady stretching surface with or without heat generation. Chamkha [47] reported the unsteady MHD heat and mass transfer on a semi-infinite vertical moving plate with heat absorption and suction/injection. The unsteady boundary layer flow induced by a stretching surface in a rotating fluid was considered by Nazar et al. [48], whereas Kumari and Nath [49] extended the above analysis to include the effect of the magnetic field. Liao [50] presented an analytic solution of unsteady boundary layer flow of Newtonian fluids induced by an impulsively stretched plate. Xu and Liao [51] considered the same problem with magnetic field and non-Newtonian fluids. In both cases, the analytic solution was obtained by using the homotopic analytic method (HAM). Kumari and Nath [52] studied the unsteady stagnation flow over a moving wall in the presence of a magnetic field. Ali and Magyari [53] investigated the unsteady flow and heat transfer caused by a submerged stretching surface whose motion is reduced gradually. The literature survey indicates that the unsteady MHD mixed convection flow over a stretching permeable surface caused by the impulsive change in surface velocity and surface temperature (heat flux) is not considered so far.

The aim of the present paper is to study the unsteady MHD mixed convection flow over an impulsively stretched vertical surface from rest in an otherwise quiescent unbounded fluid under boundary layer approximations. At the same time the surface temperature is suddenly raised from T_∞ to T_w ($T_w > T_\infty$) and then maintained at that temperature. Alternatively, a heat flux at the surface is suddenly imposed at the time t=0 which is maintained subsequently. The effect of suction/injection is included in the analysis. The non-linear coupled parabolic partial differential equations governing the unsteady mixed convection flow over a stretched surface have been solved by homotopy analysis method (HAM) as well as by Keller box finite-difference method. The results have been compared with those of Pop and Na [22], Ali and Al-Yousef [35] and Xu and Liao [51].

Using the concept of homotopy, recently Liao [54] developed a new analytic method, namely the homotopy analysis method (HAM). Unlike perturbation technique, this method does not depend upon any small or large parameters. Hence it is valid for most of non-linear problems encountered in science and engineering. In recent years, the homotopy analysis method is successfully applied to many non-linear problems [29,50,51] and the references there.

2. Analysis

Let us consider the unsteady laminar incompressible mixed convection flow of a viscous electrically conducting fluid over a heated vertical linearly stretched sheet with transverse uniform magnetic field B_0. Prior to the time t=0, the sheet has uniform temperature T_∞ and it is at rest in an unbounded quiescent fluid having constant temperature T_∞. Then at time t=0, the surface (sheet) is suddenly stretched with velocity U which varies linearly with the distance x along the surface. At the same time the surface temperature is suddenly increased from T_∞ to T_w ($T_w > T_\infty$) and subsequently maintained at that temperature. Alternatively, a heat flux q_w is imposed on the surface which is subsequently maintained. Both T_w and q_w are assumed to vary with the distance x ($T_w - T_\infty = (T_0 - T_\infty)\,(x/L)^{n_1}$, $q_w = q_0\,(x/L)^{n_1}$). The impulsive change in the surface velocity and surface temperature (heat flux) give rise to unsteadiness in the velocity and thermal fields. The stretching surface is assumed to be electrically non-conducting. The magnetic Reynolds number Re_m=$\mu_0 \sigma U L$ is assumed to be very small (where μ_0, σ, U and L denote magnetic permeability, electrical conductivity, characteristic velocity and characteristic length, respectively). Hence the induced magnetic field is negligible in comparison with the imposed magnetic field. Here no external electric field is applied and the electric field due to polarization of charges is negligible and thus $\vec{E} = 0$. For small and moderate values of the magnetic field, the Hall term can be ignored in Ohm's law as it has little effect on the flow field [56]. Therefore, only the magnetic field contributes to the Lorentz force and its component along x- direction is given by $-(\sigma B_0^2/\rho)u$. Since the velocity of the stretching surface is, generally, small in most problems, the viscous dissipation has no marked effect on the flow and heat transfer. Hence it is reasonable to neglect the viscous dissipation term in the energy equation. Here we have confined our studies to small and moderate values of imposed magnetic field ($M \le 2$). Therefore, both Ohmic heating term and Hall term in the energy equation do not have significant effect on the problem considered here. Hence they are also neglected in the energy equation. These assumptions have been made to simplify the problem. Under the above assumptions and invoking the Boussinesq approximation, the boundary layer equations based on the conservation of mass, momentum and energy governing the unsteady mixed convection flow over a stretched vertical surface with transverse magnetic field can be expressed as [19,22,35,50,55]

$$u_x + v_y = 0, \tag{1}$$

$$u_t + u u_x + v u_y = \nu u_{yy} + g\beta(T - T_\infty) - (\sigma B_0^2 u)/\rho, \tag{2}$$