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Sensitivity of the probability of failure to probability of detection curve regions

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ABSTRACT

Non-destructive inspection (NDI) techniques have been shown to play a vital role in fracture control plans, structural health monitoring, and ensuring availability and reliability of piping, pressure vessels, mechanical and aerospace equipment. Probabilistic fatigue simulations are often used in order to determine the efficacy of an inspection procedure with the NDI method modeled as a probability of detection (POD) curve. These simulations can be used to determine the most advantageous NDI method for a given application. As an aid to this process, a first order sensitivity method of the probability-of-failure (POF) with respect to regions of the POD curve (lower tail, middle region, right tail) is developed and presented here. The sensitivity method computes the partial derivative of the POF with respect to a change in each region of a POD or multiple POD curves. The sensitivities are computed at no cost by reusing the samples from an existing Monte Carlo (MC) analysis. A numerical example is presented considering single and multiple inspections.

1. Introduction

Probabilistic fatigue and fracture analysis has been used in many fields such as nuclear [1–3], petroleum [4], aircraft structures [5,6], gas turbines [7,8], and offshore structures [9] to address the uncertainty and variability of fracture mechanics parameters and NDI methods and to explicitly estimate the probability-of-failure (POF). The probabilistic analysis typically considers input parameters such as crack size, material properties, loading, geometry, and non-destructive inspection (NDI) methods. In the nuclear industry in particular, a number of probabilistic assessment codes have been developed and applied such as PRAISE, PASCAL, VISA, among others [10–14].

A number of studies have explored the benefits of in-service inspections at prescribed inspection time intervals. One conclusion is that significant reductions in probability-of-failure can be achieved, a factor of ten or more in some cases, with the probability of detection capability being the most significant factor [15–19].

Rummel et al. [20] provide a summary of a number of issues related to the application of NDE methods to structures. Quantifying the probability-of-failure (POF) provides the designer and operator an assessment of the safety and reliability of the structure with and without inspections.

A critical component of a fracture control plan is the incorporation of Non-destructive inspection methods. The effectiveness of the inspection method can be quantified numerically using a Probability-of-Detection (POD) curve, which defines the probability of detecting a defect as a function of the size of the defect. The POD curve is then incorporated in the probabilistic analysis in order to determine the reduction in the POF due to the incorporation of an inspection or multiple inspections.

This concept is well known and POD curves for a particular inspection process, material, component, etc. are typically developed through statistical experiments using seeded samples of various sizes, multiple inspectors, etc. [21,22]. For example, the "hit-miss" or "$\hat{a}$ vs. a" opportunities for a number of inspections and a number of operators are tabulated and analyzed statistically to determine the percentiles, e.g., 50 and 95% of the POD curve [23]. The percentiles are often curve fit to parametric forms such as log-logistic, lognormal, log-odds, etc.

Some common NDI methods used to identify material defects are liquid penetrant, ultrasonics, and eddy current techniques [24,25]. Each inspection technique may be described by a parametric function (POD curve). This curve provides a statistical measure to quantify the probability of detecting a crack of a certain size, length, or area. Reports discussing the derivation, application,

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Nomenclature

POF	probability-of-failure
POD	probability of detection
q	inspection number
t	time
t_q	time of inspection q
$\boldsymbol{x}$	all random variables
$\boldsymbol{y}$	all random variables that affect crack size a
$\boldsymbol{\theta}$	parameters of POD curve
a_i	initial crack size
$a(\boldsymbol{y}, t_q)$	crack size at time t_q
$f_{\boldsymbol{x}}(\boldsymbol{x})$	joint PDF of random variables
$I(\boldsymbol{x}, t)$	indicator function that denotes structural failure
N	total number of Monte Carlo samples
N_q	number of Monte Carlo samples that reach inspection q and are not detected
$P_f(t)$	probability-of-failure at time t
$POD_q(\boldsymbol{\theta}, a(\boldsymbol{y}, t_q))$	probability of detection curve for inspection q
$CPOD_q(\boldsymbol{\theta}, a(\boldsymbol{y}, t_q))$	complementary POD_q, equals 1 minus POD_q
$\frac{\partial P_f(t)}{\partial_\theta}$	sensitivity of the probability-of-failure with respect to the POD parameters $\boldsymbol{\theta}$
$\Omega_q(\boldsymbol{\theta}, a(\boldsymbol{y}, t_q))$	$-\frac{\partial POD_q}{\partial \theta} \frac{1}{CPOD_q}$

and limitations of POD curves have been published [26].

Development of a comprehensive fracture control plan requires a comparison of the effectiveness of various NDI methods vs. cost. Although trade-off studies can clearly be accomplished using repeated "what-if" analyses, a more effective approach is to provide sensitivity information from a single analysis such that the analyst can quickly estimate the effect of a change in the POD curve.

An additional consideration is that it is likely that only a particular "region" (left tail, center, right tail) of the POD curve is effective at reducing the POF, but this information is not available from what-if analyses. Therefore, a method to compute sensitivity information that will identify the region of the POD curve most effective at reducing the POF is derived and demonstrated.

Previous related work has been done on developing sensitivities with respect to the parameters of a POD curve, e.g., mean and standard deviation [27]. This information is useful in that the effect of small changes in the POD curve on the POF can be quickly assessed without further probabilistic analyses. This approach treats the POD curve parametrically. That is, the POD curve is defined using a parametric equation, such as a lognormal distribution, then the sensitivity with respect to the parameters of the lognormal distribution are computed. In contrast, the non-parametric local sensitivity method presented here can discern the importance of a particular region of a POD curve (left tail, center, right tail), etc. Using the localized sensitivities allows an operator to compare two POD curves in their "important region", and determine the most advantageous POD curve for the problem at hand. This capability provides an additional degree of freedom to select the optimum NDI method and to design an optimum POD curve for a particular application.

The key concept behind the local sensitivity method is to compute the partial derivative of the POF, $P_f(t)$, with respect to a value of the POD curve at position j along the POD curve, see Fig. 1, i.e., $\partial P_f(t)/\partial\theta^j$. This is repeated at multiple positions along the POD curve in order to assess the relative sensitivity of the POD with respect to different regions of the POD curve (left tail, center region, right tail). These partial derivatives provide a first order estimate of the changes in the POF with respect to changes in the parameters of the POD curves. The methodology used is derived from the Score Function method [28–30].

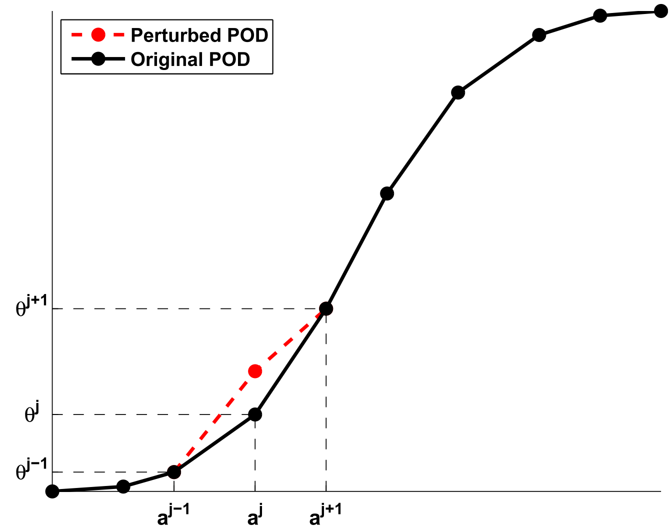

Fig. 1. Schematic of Localized sensitivity method on the POD curve.

Using these sensitivities, the change in the POF for a given change in a single POD parameter θ^j can be approximated as $\Delta P_f(t) = \frac{\partial P_f(t)}{\partial\theta^j}\Delta\theta^j$ and for multiple parameters as $\Delta P_f(t) \approx \sum_{j=1}^{J}\frac{\partial P_f(t)}{\partial\theta^j}\Delta\theta^j$. Thus, the computed sensitivities provide a convenient method to estimate the amount of change in the POF for a prospective change in the POD parameters.

The beauty of the formulation as shown below is that existing samples from a probabilistic fatigue and fracture analysis using MC sampling can be reused to compute the sensitivities. As a result, the sensitivities can be obtained for negligible cost. In addition, the discretization of the POD curve into local regions is arbitrary, at the discretion of the user, and different discretizations can be analyzed without generating new samples. The accuracy of the results depends, of course, on the number of MC samples used; therefore, variance estimates have been derived.

The paper is organized as follows. The methodology is described in Section 2. The analysis procedure is summarized in Section 3. Numerical examples are described in Section 4, followed by the Conclusions in Section 5. The Appendices contain more thorough mathematical details, discussion of the critical new parameter, Ω, and variance estimates of the sensitivities.

2. Methodology for probability-of-failure sensitivities

The probability-of-failure, P_f, as a function of time for fatigue crack growth analysis without inspections is evaluated as

$$P_f(t) = \int_{g(\boldsymbol{x},t)\le 0} f_{\boldsymbol{x}}(\boldsymbol{x})d\mathbf{x} \tag{1}$$

where t is the time in cycles or hours and $\boldsymbol{x} = [x_1, x_2, x_3, \ldots, x_n]^T$ is a vector of n random variables, e.g., initial crack size, fracture toughness, loading, and crack growth parameters, and $f_{\boldsymbol{x}}(\boldsymbol{x})$ denotes the joint probability density function. The limit state is defined as $g(\boldsymbol{x}, t)$ with failure defined as $g(\boldsymbol{x}, t) \le 0$. For example, to compute the POF at an observation time, t_0, the limit state is $g(\boldsymbol{x}, t) = t_f(\boldsymbol{x}) - t_0$, where t_f is the cycles-to-failure. The cycles-to-