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A finite element study of the deformations, forces, stress formations, and energy losses in sliding cylindrical contacts

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Abstract

This work presents the results of a finite element analysis (FEA) used to simulate two-dimensional (2D) sliding between two interfering elasto-plastic cylinders. The material for the cylinders is modeled as elastic-perfectly plastic and follows the von Mises yield criterion. The FEA provides trends in the deformations, reaction forces, stresses, and net energy losses as a function of the interference and sliding distance between the cylinders. Results are presented for both frictionless and frictional sliding and comparisons are drawn. The effects of plasticity and friction on energy loss during sliding are isolated. This work also presents empirical equations thatt relate the net energy loss due to sliding under an elasto-plastic deformation as a function of the sliding distance. Contour plots of the von Mises stresses are presented to show the formation and distribution of stresses with increasing plastic deformation as sliding progresses. This work shows that for the plastic loading cases the ratio of the horizontal force to the vertical reaction force is non-zero at the point where the cylinders are perfectly aligned about the vertical axis. In addition, a “load ratio” of the horizontal tugging force to the vertical reaction force is defined. Although this is analogous to the common definition of the coefficient of friction between sliding surfaces, it just contains the effect of energy loss in plasticity. The values of the contact half-width are obtained for different vertical interferences as sliding progresses.

Keywords: Cylinders; Finite element analysis; Frictional sliding; Frictionless sliding; Plasticity; Deformation; Stress; Reaction force; Energy loss

1. Introduction

Sliding contact between two elasto-plastic cylinders and spheres has important engineering applications in both the macro- and the microscale. The current results are normalized to be valid in both scales as long as continuum mechanics prevails. In microscale, it is well known that asperities deform plastically during sliding contact between rough surfaces. Thus, it is important to know the effect the contact has on the surface material and the geometry through plastic deformations and residual stresses. In macroscale, this information may be useful in analyzing the friction, wear, and deformation of contacts such as in gears, rolling element bearings, wheel on rail, when sliding may occur (in addition to rolling). In an electromagnetic launcher (EML) [1] an armature slides in a predefined spacing between two rails and, hence, this application lends itself specifically to the boundary conditions used in the current work. The results presented here may also be valuable in analyzing human joints, such as that investigated by Chen et al. [2], wherein 2D plane strain finite elements are employed to model the temporomandibular joint using hyperelastic (Mooney–Rivlin) material. The approach is similar to the one taken in the current study only that here metallic-like material behavior is prevailing.

Both elastic and elastic–plastic spherical contacts have been analyzed in great detail in the last four decades. Predominantly considering normal loading only, a wide array of works have analyzed the contact of rough surfaces as reviewed by Liu et al. [3]. These works are based on the contact behavior of a single asperity in a statistical model of multiple asperity contact. All these works, share the common methodology of Thomas [4] and Greenwood [5] that is as follows:

(1) Replacing the two rough surfaces by a smooth surface in contact with an equivalent rough surface.

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Nomenclature

b	contact half width
C	critical yield stress coefficient
E	elastic modulus
E'	equivalent modulus of elasticity, $\frac{1}{E'} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$
F_x	horizontal reaction force at the base of the bottom cylinder
F_y	vertical reaction force at the base of the bottom cylinder
i	load step number
L	length of contact
n	number of load steps employed to simulate a quasi-static sliding process
P	contact force
P^*	non-dimensional load, P/P_c
p_o	maximum contact pressure
R	radius of the cylinder
R	equivalent radius, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
S_y	yield strength
U	potential (strain) energy
u	maximum vertical displacement
x	horizontal sliding distance covered by the top cylinder up to the ith load step
Δx	total horizontal distance covered by the top cylinder to complete sliding
δx	equal increments in which the total sliding horizontal sliding is covered
ν	Poisson's ratio
σ_e	maximum equivalent von Mises stress
ω	interference between cylinder surfaces
ω^*	non-dimensional vertical interference between cylinders, ω/ω_c

Superscript

$*$	dimensionless

Subscripts

c	critical value at onset of plastic deformation
$'$	equivalent
1	bottom cylinder
2	top cylinder
net	net value after sliding is completed
res	residual value after sliding is completed
x	corresponding to horizontal axis
y	corresponding to vertical axis

(2) Replacing asperities with simple geometrical shapes.
(3) Assume a probability distribution for asperity parameters.

Some of these works are restricted mainly to pure elastic regime, such as the landmark work by Greenwood and Williamson [6]. Other works, such as Greenwood and Tripp [7], Lo [8], Whitehouse and Archard [9], Tsukizoe and Hisakado [10], and Bush et al. [11,12], extend the Greenwood and Williamson model in the elastic regime to a variety of geometries and different basic assumptions. Other works concentrate on pure plastic deformation, and are based on the models of Abbott and Firestone [13] and Tsukizoe and Hisakado [10].

Normal spherical contacts in the elastic–plastic regime by Evseev et al. [14], Chang [15], and Zhao [16]. FEA has been used by Vu-Quoc et al. [17] to analyze contact between two spheres, which by symmetry is equivalent to that of one sphere in contact with a frictionless rigid plane, but the analysis is restricted to specific parameters and lack generality. Adams and Nosonovsky [18] provide a review on contact modeling with an emphasis on the forces of contact and their relationship to the geometrical, material and mechanical properties of the contacting bodies.

Recently, Jackson and Green [19], Wang and Keer [20], and Nelias et al. [21], have explored hemispherical elastic–plastic contact in normal loading. However, the characteristics of normal contact as opposed to sliding contact are quite different, and thus the latter is explored in this work. Hamilton and Goodman [22] presented implicit equations and graphs of yield parameter and tensile stress distribution for circular sliding contact using the von Mises criterion for the prediction of yielding. Hamilton [23] further developed the implicit results in [22] to obtain explicit formulae for the stresses beneath a sliding, normally loaded Hertzian contact. However, these studies [22,23] concentrated on the effect of increasing friction in a sliding contact against a rigid flat, and on the resulting development of impending failure regions, but a coefficient of friction had a priori been imposed. In contrast, this works isolates the effects of purely frictionless sliding of interfering cylinders, and hence the development of stresses, energy loss, and other phenomena occur solely due to mechanical deformation. In [24] a dynamic analysis gives an estimation of the contact forces between wheels and rails in sliding. The analysis herein, is naturally related to such a line (or cylindrical) contact. Perhaps one of the earliest attempts in tackling interference sliding between two bodies (spheres) is that by Faulkner and Arnell [25], who quote extremely long execution times even for very coarse FEA meshes (~960 h for each simulation), leaving out generalization of the results. Steady-state dry frictional sliding between two elastic bodies by using Fourier series and integral transform techniques has been examined by Nosonovsky and Adams [26].

It is clear from the literature survey that a thorough investigation of the actual forces, deformations, stress formations, and most importantly energy losses due to plasticity for sliding in