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# Geometrically non-linear analysis of laminated composite structures using a 4-node co-rotational shell element with enhanced strains

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**Abstract**

To demonstrate the solutions of linear and geometrically non-linear analysis of laminated composite plates and shells, the co-rotational non-linear formulation of the shell element is presented. The combinations of an enhanced assumed strain (EAS) in the membrane strains and assumed natural strains (ANS) in the shear strains improve the behavior of 4-node shell element. To secure computational efficiency in the incremental non-linear analysis, the present element uses the form of the resultant forces pre-integrated through the thickness. The transverse shear stiffness of the laminates is defined by an equilibrium approach instead of the shear correction factor. Numerical examples of this study show very good agreement with the references.


*Keywords:* Non-linear behavior; Laminates; Enhanced assumed strain (EAS); Assumed natural strain (ANS); Co-rotational resultant shell element

## 1. Introduction

Due to their high specific modulus compared to conventional material, fiber-reinforced composite materials are increasingly being used in various structural applications. Fiberglass, developed in the late 1940s, was the first modern composite and is still the most commonly used material. Composite materials are formed by combining two or more types of material that has quite different properties. The combination of the different materials gives the composite unique properties. In fiberglass, the reinforcement is fine threads or fibers of glass, which is often woven into a sort of cloth, and the matrix is a plastic. The threads of glass in fiberglass are very strong under tension but they are also brittle and snap if bent sharply. Any deformation of a sheet of fiberglass necessarily stretches some of the glass fibers, but they are able to resist deformation, as the result, even a thin sheet is very strong. It is also quite light, which is an advantage in many applications. By choosing an appropriate combination of reinforcement and matrix material, manufacturers can produce properties that exactly fit the requirements of a particular structure for a particular purpose. Composites will not completely replace traditional materials such as steel, but in many cases they satisfy the manufacturers' needs.

The finite element modeling of shell structures focusing on computational aspects can be traced back to Ahmad et al. [1]. While such elements are capable of dealing with thick plate and shell problems, their performance deteriorates rapidly as the element thickness becomes thin, which is called shear locking. The efforts by many investigators have been directed at overcoming the transverse shear locking problem in Mindlin–Reissner type elements. Solutions originally proposed to alleviate locking are reduced integration or selective integration. Another type of solution, which is adopted for the present element, is the assumed natural strain (ANS) and enhanced assumed strain (EAS) method. The early use of the ANS method can be traced to MacNeal [2] and Dvorkin and Bathe [3]. Kim and Park [4] used ANS method to formulate an 8-node element. The results obtained are comparable and in some cases are better to those presented by Bathe and Dvorkin [5]. Kim et al. [6] and Park et al. [7] presented quasi-conforming formulation of 4-node stress resultant shell elements. The EAS method was studied by Andelfinger and Ramm [8], César de Sá et al. [9] and Fontes Valente et al.

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**Nomenclature**

| | |
|---|---|
| Bar over (-) | value measured at mid-surface |
| $x, y, z$ | global coordinate system |
| $r, s, t$ | local coordinate system |
| $\xi, \eta, \zeta$ | natural coordinate system |
| $i$ | superscript referring to node number $i$ |
| $\mathbf{J}$ | Jacobian matrix |
| m, b, $q$ | subscript referring to membrane, bending and transverse shear, respectively |
| $\mathbf{P}$ | position vector |
| $H^i$ | shape function at node $i$ |
| $\mathbf{T}$ | direction cosine of the new local axes with respect to the global axes |
| $\bar{\mathbf{U}} = (\bar{U}, \bar{V}, \bar{W})$ | global translation of the mid-surface |
| $\boldsymbol{\theta} = (\theta_x, \theta_y, \theta_z)$ | global rotation of the mid-surface |
| $\bar{\mathbf{u}} = (\bar{u}, \bar{v}, \bar{w})$ | local translation of the mid-surface |
| $\boldsymbol{\varphi} = (\varphi_r, \varphi_s, \varphi_t)$ | local rotation of the mid-surface |
| $\mathbf{U} = (\bar{\mathbf{U}}, \boldsymbol{\theta})$ | global displacement |
| $\mathbf{u} = (\bar{\mathbf{u}}, \boldsymbol{\varphi})$ | local displacement |
| $\mathbf{V}_r, \mathbf{V}_s, \mathbf{V}_t$ | base vector tangential to the local coordinates |
| $\mathbf{e}_\mathrm{m}, \mathbf{e}_\mathrm{b}, \mathbf{e}_q$ | linear part of the membrane, bending and transverse shear strain vector |
| $\gamma_{\xi\varsigma}, \gamma_{\eta\varsigma}$ | transverse shear strains in the natural coordinates |
| $\alpha$ | enhanced strain parameters |
| $\mathbf{N}, \mathbf{M}, \mathbf{Q}$ | resultant membrane forces ($N_r, N_s N_{rs}$), moments ($M_r, M_s M_{rs}$), and transverse shear forces ($Q_r, Q_s$) |
| $\mathbf{M}_\xi, \mathbf{M}_\mathrm{E}$ | interpolation function of enhanced assumed strain field in isoparametric and physical space, respectively |
| $\mathbf{A}, \mathbf{B}, \mathbf{D}, \bar{\mathbf{A}}$ | membrane, membrane-bending, bending and transverse stiffness matrix, respectively |
| $\bar{\mathbf{C}}^k_{ij}$ | transformed reduced stiffness matrix |
| $\mathbf{K}^\mathrm{e}$ | element stiffness matrix |
| $S, V$ | surface area and volume of the element |
| $G_{13} G_{23}$ | transverse shear modulus in the plane 1–3 and 2–3 |
| $v$ | Poisson's ratio |

[10] to improve the behavior of the shell element. Andelfinger and Ramm [8] developed the four-node membrane, plate and shell elements and eight-node solid elements by using the EAS method proposed by Simo and Rifai [11]. César de Sá et al. [9] proposed an alternative formulation based on the EAS method of Simo and Rifai [11] to avoid shear locking effects in four-node shell element. Fontes Valente et al. [10] extended the EAS finite element formulation in order to account for geometrical and material non-linearities.

Previously, the transverse shear stiffness for Mindlin type laminate composite plates and shells have been treated and studied as similar conditions as isotropic materials, i.e. the transverse shear stiffness were multiplied with the same factor as that used for an isotropic material, e.g. Yang et al. [12] and Whitney and Pagano [13]. This idea has also been used in the finite element method. Wittrick [14] showed that the displacement mode is required to obtain an effective transverse shear modulus. The work of Rolfes and Rohwer [15] assumes two cylindrical bending modes for an improved transverse shear modulus based on the first order shear deformation theory for finite elements.

The four-node shell element with ANS converges slowly and gives less accurate solutions to problems that include membrane behavior. One method to improve these behaviors is to add proper enhanced strain terms in the membrane strains, as shown by Andelfinger and Ramm [8]. The objective of this paper is to present the co-rotational formulation of a non-linear 4-node shell element based on the EAS and ANS methods. For laminated composites, instead of using the conventional transverse shear stiffness with correction factor, the equilibrium approach suggested by Rolfes and Rowher [15] is used. The geometrically non-linear formulation of the shell element is based on Mindlin–Reissner theory, assuming small strains and large rotations. In order to define the motion of the element and remove the rigid body translations and rotations, the co-rotational method by Kim and Lomboy [16] is employed. The displacement field is referred to a set of local co-rotational co-ordinates. Thus, the deformation can be isolated by removing the rigid body rotation from the total nodal displacements. Also, the geometric stiffness is analytically integrated through the thickness. In comparison with volume integration in laminated composites, which are generally used in degenerated shell elements, the computational time is significantly reduced for non-linear analysis of shell structures.

## 2. Geometry of the shell element

The 4-node shell elements shown in Fig. 1 indicates the relation between local $(r, s, t)$ and natural curvilinear co-ordinate $(\xi,\eta,\zeta)$ established by base vectors. The mid-surface shows two non-dimensional coordinates $\xi, \eta$ while the axis $\zeta$ shows normal to the shell mid-surface. The origin of the curvilinear co-ordinate system is set to the center of each element. In general, this coordinate system is not orthogonal. Hence, the local orthogonal coordinate system $(r, s, t)$ at the center of the element is constructed. The $(r, s)$ and $(\xi,\eta)$ surface are coplanar.

The base vector $(\mathbf{V}_r, \mathbf{V}_s, \mathbf{V}_t)$ which are tangential to the local co-ordinates shown in Fig. 1 are defined as follow:

$$\mathbf{V}_t = \frac{\bar{V}_\xi \times \bar{V}_\eta}{|\bar{V}_\xi \times \bar{V}_\eta|}, \quad \mathbf{V}_s = \frac{(V_t \times \overline{V}_\xi) + \overline{V}_\eta}{|(V_t \times \overline{V}_\xi) + \overline{V}_\eta|}, \quad \mathbf{V}_r = \mathbf{V}_s \times \mathbf{V}_t. \tag{1}$$