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Objective comparison of the Unified Curve and Master Curve methods

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ABSTRACT

The Unified Curve and the Master Curve are two popular cleavage fracture toughness assessment engineering methods. The methods are very similar. They basically differ only in the assumed fracture toughness temperature dependence. The standard Master Curve approximates the temperature dependence as being fixed, whereas the Unified Curve assumes that the shape changes as a function of transition temperature. The shape difference becomes significant only for highly brittle steels. Previous comparisons of the two methods have applied a procedure that may cause a bias on the comparison when assessing censored data sets. Here, a fully objective comparison using the censored likelihood, have been made for 50 large data sets with transition temperatures in the range +8 °C … +179 °C. The standard Master Curve shows overall a trend of higher likelihood than the Unified Curve. It is also shown that, because of shortages connected to the use of the Prometey probabilistic cleavage fracture model, the Unified Curve cannot be considered universally applicable.

1. Introduction

The Master Curve (MC) method for the description of brittle fracture toughness [1], which forms the test standard ASTM E1921 [2], is included among others in the ASME code [3,4] and the structural integrity standard BS7910 [5]. The MC has been widely validated for numerous different structural steels [6–9]. The MC can be divided into a theoretical and an empirical part. The theoretical part, derived based on statistical modelling of the cleavage fracture event, gives the scatter of fracture toughness as a function of specimen thickness (crack front length) and median fracture toughness. This part of the standard MC has the form of Eq. (1). B is the specimen thickness (crack front length). B_0 is a normalisation thickness taken as 25 mm. K_0 corresponds to a cumulative failure probability of 63.2% and is related to the median fracture toughness ($K_{0.5}$) through Eq. (2). The mean fracture toughness is slightly lower than the median, because the fracture toughness distribution is unsymmetrical. The constant in Eq. (2) changes for the mean fracture toughness to 0.906. K_{min} represents a lower limiting stress intensity factor, below which cleavage crack propagation is impossible on a micro-scale [10].

$$P_f = 1 - \exp\left\{ -\frac{B}{B_0} \cdot \left(\frac{K_I - K_{min}}{K_0 - K_{min}} \right)^4 \right\} \tag{1}$$

$$K_{0.5} = 0.912 \cdot (K_0 - K_{min}) + K_{min} \tag{2}$$

The empirical part of the standard MC is related to the temperature dependence of K_0, in accordance of Eq. (3), by which the fracture toughness is expressed in the form of a single reference temperature, T_0. It corresponds to the temperature where a 25 mm thick specimen has a mean fracture toughness of 100 MPa$\sqrt{m}$. The dependence is based on a best fit to a number of large data sets available at the time. The original data is reproduced in Fig. 1 [11]. The T_0 values of the original data cover a temperature range from −109 °C to +51 °C. The exponential shape of the temperature dependence comes from the assumption that the events controlling cleavage fracture are thermally activated and as such should follow an exponential shape.

$$K_0^{MC} \approx 31 + 77 \cdot \exp(0.019 \cdot [T - T_0]) \quad \text{MPa}\sqrt{m}, \ ^\circ C \tag{3}$$

The standard MC temperature dependence does not contain the assumption that all structural steels would follow Eq. (3). There are too many factors affecting the temperature dependence to assume a universally constant behaviour for all structural steels [10]. The ASTM E1921 temperature dependence given by Eq. (3) is only an approximation to fracture toughness data in the T_0 region −50 °C

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Notation

d	initiator size
$\bar{d}$	mean initiator size
d_c	critical initiator size
d_n	Weibull normalisation parameter
e_{eq}	equivalent strain
f	probability density
h	material constant
m	Weibull exponent
m_0	material constant
n	number of volume elements
r	number of un-censored data
x	distance
A_d	material constant
B	specimen thickness or crack front length
B_0	normalising thickness = 25 mm
C_1	material constant
C_2	material constant
E'	modulus of elasticity in plane strain
K_I	stress intensity factor
K_{JC}	fracture toughness
K_{min}	lower limiting fracture toughness
K_0	fracture toughness corresponding to a cumulative failure probability of 63.2%
K_{0i}	K_0 for initiation
$K_{0.5}$	median fracture toughness
L	likelihood
MC	Master Curve
N	number of initiators in volume element
$\bar{N}_V$	average number of initiators in volume element V
P_f	cumulative probability
$P(K_\infty)$	probability of cleavage crack propagation in a unified stress field
$P\{d\}$	initiator size distribution
$\Pr\{I\}$	cleavage initiation probability
$\Pr\{I/O\}$	conditional probability of cleavage initiation with no prior void initiation at the site
$\Pr\{V/O\}$	probability of having void initiation
S	survival probability = $1-P_f$
S_c	critical brittle fracture stress
S_0	material constant
T	temperature
T_0	temperature where a 25 mm thick specimen has a mean fracture toughness of 100 MPa$\sqrt{m}$
U	normalised distance
UC	Unified Curve
V	volume
η	Weibull exponent
δ	censoring parameter
γ_p	effective surface energy
ν	material constant
θ	angle
σ_d	initiator strength
$\tilde{\sigma}_d$	Weibull normalisation parameter
σ_{d0}	minimum cleavage initiator strength equal to σY at 0 K
σ_{eq}	equivalent stress
σ_Y	yield strength
σ_{YG}	athermal part of yield strength
σ_{yy}	crack opening principal stress
σ_{part}	stress acting in the initiator particle
σ_0	Weibull normalisation parameter
ΔB	thickness increment
Δx	distance increment
ΔU	normalised distance increment
$\Delta\theta$	angle increment
Ω	parameter in the Unified Curve

… +50 °C. For applications outside this region, the MC methodology [10] advises to perform tests at the relevant temperature.

Another method for the description of brittle fracture toughness claiming to provide a theoretical description of the temperature dependence has also been proposed [12]. The method is known as the Unified Curve (UC) and has been included in the Russian nuclear safety code [13]. It is based on a theoretical cleavage fracture model known as the Prometey probabilistic model [12]. The model has changed somewhat since the development of the UC [14,15], but the UC is still based on the form of the model in Ref. [12]. The scatter and size effect in the UC is taken identical to the MC in the form of Eq. (1). The only difference lies in the temperature dependence of K_0. The UC temperature dependence, expressed for K_0, has the form of Eq. (4) [12].

$$K_0^{UC} \approx 26.6 + 1.096 \cdot \Omega \cdot \left(1 + \tan h\left(\frac{T-130}{105}\right)\right) \quad \text{MPa}\sqrt{\text{m}}, \ ^\circ\text{C} \tag{4}$$

The median fracture toughness at 130 °C equals simply the sum 26 MPa$\sqrt{m}$ + Ω MPa$\sqrt{m}$.

The standard MC and the UC can simply be compared by normalising the UC to coincide with the MC at T_0 (Fig. 2). For Ω values above 500 MPa$\sqrt{m}$, the UC is basically undistinguishable from the MC.

Mathematically it is simple to derive a relation between T_0 and Ω. The relation (Eq. (5)) is somewhat dependent on the toughness level used for the derivation, but the trend is clear as seen from Fig. 3. There is an inverse relation between the parameters (T_0 and Ω) and only for $T_0 > 0$ °C, decreases Ω below 500 MPa$\sqrt{m}$. In order to see any statistically significant differences, a comparison between the two methods must thus be based on materials with $T_0 > 0$ °C.

$$\Omega = \frac{K_{JC} - 26}{1 + \tan h\left\{\frac{\frac{\ln\left(\frac{K_{JC}-30}{70}\right)}{0.019} + T_0 - 130}{105}\right\}} \text{MPa}\sqrt{\text{m}}, \ ^\circ\text{C} \tag{5}$$

Previous comparisons of the UC and the standard MC have been made with the parameters δ and σ [16]. These parameters basically estimate the resulting standard deviation between the predicted mean fracture toughness (from MC or UC) and individual fracture toughness values or with an estimate of the mean fracture toughness value at a specific temperature. The latter method would be better if a sufficient number of tests would be available at one temperature. Both parameters may, however, be somewhat subjective when used with censored data sets.

One way of making an objective comparison between the standard MC and UC temperature dependencies for censored data sets is to use the maximum likelihood method, allowing for proper censoring.