

Non-linear modal properties of non-shallow cables

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Abstract

A non-linear mechanical model of non-shallow linearly elastic suspended cables is employed to investigate the non-linear modal characteristics of the free planar motions. An asymptotic analysis of the equations of motion is carried out directly on the partial-differential equations overcoming the drawbacks of a discretization process. The direct asymptotic treatment delivers the approximation of the individual non-linear normal modes. General properties about the non-linearity of the in-plane modes of different type—geometric, elasto-static and elasto-dynamic—are unfolded. The spatial corrections to the considered linear mode shape caused by the quadratic geometric forces are investigated for modes belonging to the three mentioned classes. Moreover, the convergence of Galerkin reduced-order models is discussed and the influence of passive modes is highlighted.

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1. Introduction

The linear and non-linear dynamics of suspended elastic cables have received considerable attention due to their use in several applications in the fields of communications, electricity, mooring systems, transportation, and crane-operation systems. The linear vibration theory of suspended cables is attributed to the work of Irvine and Caughey [1], Irvine [2], and Triantafyllou and co-workers [3,4]. The modal properties of shallow cables have been shown to depend on one elasto-geometric parameter, the so-called Irvine's parameter.

Free non-linear vibrations were studied in [5,6] to mention only a few works. Several studies dealt either theoretically or experimentally with harmonically forced oscillations both for the non-resonant and the resonant cases, the latter including a plethora of modal interactions. In particular, it has been demonstrated that the responses of shallow suspended cables near the first crossover exhibit complex behavior due to the presence of multiple internal resonances involving in-plane and out-of-plane modes. Examples include the coexistence of different

types of periodic motions and the occurrence of quasiperiodic and chaotic oscillations [7–10].

The great majority of the works, especially those addressing non-linear vibrations, deals with shallow cables described by approximate mechanical models based on the static condensation of the longitudinal dynamics. In particular, a characterization of the non-linear normal modes of shallow cables has been addressed in [11,12]. An extensive and updated review of the state of the art on shallow cables can be found in [10].

On the contrary, a few studies have addressed linear and non-linear dynamic behaviors of non-shallow cables whereas non-shallow configurations may occur in a number of engineering applications such as in cables used for cable railways, transmission lines, mooring lines or tag-lines. Hence, there is a practical and theoretical interest in investigating non-shallow cable configurations and the leading dynamics around them.

In [13,14], a non-linear mechanical model of non-shallow cables, describing the fully coupled longitudinal and transverse dynamics, was presented. Therein, results of the investigations into the spectral properties of linear free vibrations around the catenary configurations were reported. Differently from shallow cables, whose linear dynamics depend solely on Irvine's parameter, it is shown that the linear vibration properties of non-shallow cables depend on two parameters

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separately regulating the cable elastic and geometric stiffnesses. Among other properties, it was also determined where, in parameter space, the three classes of modes appear, namely, geometric, elasto-static and elasto-dynamic modes which were already partly mentioned in [3]. It was demonstrated that the elastic modes belong to a complete sequence of symmetric and skew-symmetric stretching modes starting from the lowest elasto-static stretching mode with nearly constant elongation. These modes are manifested in the neighborhood of the various crossovers, the well-known (elasto-static) lowest crossovers and the highlighted higher-order (elasto-dynamic) crossovers. Away from these crossovers, the modes are geometric modes, in the sense that they are prevalently governed by the geometric stiffness, and exhibit leading transverse displacements.

In this paper, the primary focus is on the effects of the geometric non-linearity on the modal properties; in particular, the objective is to characterize the non-linear properties of the individual in-plane modes with a clear effort towards unfolding general properties. Further, the a priori knowledge of the non-linear modal properties is the basis for the prediction of the features of unimodal forced responses and interaction phenomena (the type of bifurcations, the possible routes to chaotic solutions, ...).

The non-linear partial-differential equations of motion and boundary conditions are recast in first-order form. Then, the asymptotic analysis of individual non-linear normal modes is presented. The main results on the non-linear characteristics of the modal motions are summarized. They mostly relate to the so-called effective non-linearity coefficient of the considered mode which regulates the bending of the backbone and to the shape functions dictating the spatial corrections to the considered linear mode shape at second order. General conclusions are drawn about the non-linear laws of the modes, depending on whether they are geometric, elasto-static or elasto-dynamic.

2. Equations of motion

We denote $(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$ the orthonormal basis of a fixed inertial reference frame with origin in O (Fig. 1), let N_0^* and N^* describe the static axial force due to gravity in the initial configuration $\mathcal{C}_0$ and the incremental dynamic force arising in the change from the initial to the current configuration $\mathcal{C}$. Imposing the balance of linear and angular momentum yields the equation of free undamped motions as

$$\frac{\partial[N_0^*(\mathbf{a} - \hat{\mathbf{a}}_0)]}{\partial x^*} + \frac{\partial(N^*\mathbf{a})}{\partial x^*} = m_0 \sec \theta_0 \frac{\partial^2 \mathbf{u}^*}{\partial t^{*2}}, \quad (1)$$

where x^* indicates the horizontal coordinate along the fixed $\mathbf{i}$ direction. In Eq. (1), m_0 is the mass per unit cable length in its initial configuration $\mathcal{C}_0$ lying in the $(\mathbf{i}, \mathbf{j})$ -plane—here expressed as $\hat{\mathbf{p}}_0^*(x^*) = x^*\mathbf{i} + y^*(x^*)\mathbf{j}$; $\mathbf{u}^*$ is the displacement vector, henceforth conveniently decomposed as $\mathbf{u}^*(x^*, t^*) = \hat{\mathbf{u}}^* + w^*\mathbf{k}$ with $\hat{\mathbf{u}}^*$ being the in-plane displacement (i.e., lying in the $(\mathbf{i}, \mathbf{j})$ -plane); $\hat{\mathbf{a}}_0(x^*)$ is the unit tangential vector in $\mathcal{C}_0$ and $\theta_0(x^*)$ is the angle between $\hat{\mathbf{a}}_0$ and $\mathbf{i}$ given by $\theta_0(x^*) = \arctan(dy^*/dx^*)$; $\mathbf{a}$ is the unit vector tangent to the current

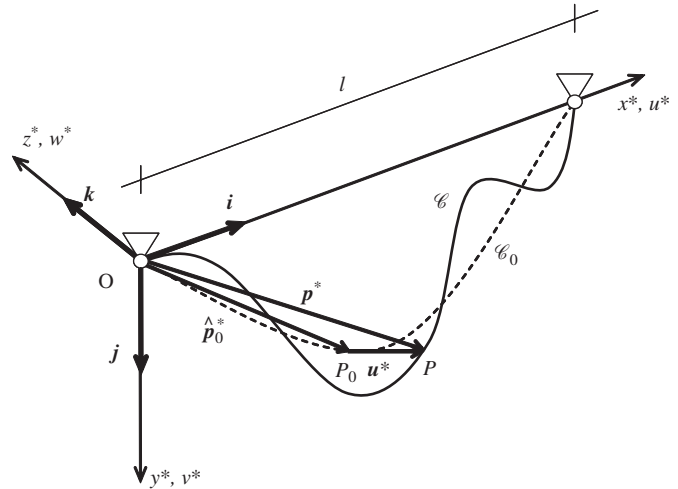


Fig. 1. The geometry of the cable model with the inertial reference frame: $\mathcal{C}_0$ and $\mathcal{C}$ indicate the initial static and the current dynamic configurations, respectively.

configuration of the cable axis. Henceforth, the same notation will be employed throughout the manuscript, namely, lower-case bold italic letters indicate vectors in E^3 , hatted lowercase bold italic letters denote vectors lying in the $(\mathbf{i}, \mathbf{j})$ -plane, and lowercase bold italic letters denote the corresponding algebraic vectors.

The cable axial stretching associated with the deformation from $\mathcal{C}_0$ to $\mathcal{C}$ is

$$v = \left| \frac{d\mathbf{p}^*}{ds^*} \right| = \cos \theta_0 \left| \frac{d\mathbf{p}^*}{dx^*} \right| = \cos \theta_0 \sqrt{(1 + u')^2 + (\tan \theta_0 + v')^2 + w'^2}, \quad (2)$$

where $|\cdot|$ represents the magnitude of the vectorial argument, s^* is the arclength along the cable axis in $\mathcal{C}_0$ and the prime indicates differentiation with respect to the non-dimensional coordinate $x = x^*/\ell$. Further, the displacement components are non-dimensionalized as $u := u^*/\ell$, $v := v^*/\ell$, $w := w^*/\ell$. The unit vector in the current tangential direction is expressed as

$$\mathbf{a} := \frac{\mathbf{p}'}{|\mathbf{p}'|} \equiv \cos \theta_0 \frac{(1 + u')\mathbf{i} + (\tan \theta_0 + v')\mathbf{j} + w'\mathbf{k}}{v}. \quad (3)$$

Due to the relatively high axial stiffness of typical engineering cables, the initial configuration of the cable, represented by the catenary, and the axial load are, respectively,

$$y(x) = \frac{1}{\gamma} \left[\cosh \frac{\gamma}{2} - \cosh \gamma \left(\frac{1}{2} - x \right) \right], \quad N_0(x) = \cosh \gamma \left(\frac{1}{2} - x \right), \quad (4)$$

where $\gamma := y^*/\ell$, $\gamma := mgl/H_0^*$ is solution of the geometric compatibility condition $\sinh(\gamma/2) = (\gamma/2)\eta_0$ with $\eta_0 := L_0/\ell$, L_0 is the initial total length of the cable, $N_0(x) := N_0^*(x^*)/H_0^*$, and H_0^* is the horizontal projection of N_0^* . The extensibility of the cable under its own weight is neglected as it is typically done

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