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Plastic limit pressure of spherical vessels with combined hardening involving large deformation

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A B S T R A C T

The paper aims to investigate plastic limit pressure of spherical vessels of nonlinear combined isotropic/kinematic hardening materials. The Armstrong-Frederick kinematic hardening model is adopted and the Voce hardening law is incorporated for isotropic hardening behavior. Analytically, we extend sequential limit analysis to deal with combined isotropic/kinematic hardening materials. Further, exact solutions of plastic limit pressure were developed analytically by conducting both static and kinematic limit analysis. The onset of instability was also derived and solved iteratively by Newton's method. Numerically, elastic–plastic analysis is also performed by the commercial finite-element code ABAQUS incorporated with the user subroutine UMAT implemented with user materials of combined hardening. Finally, the problem formulation and the solution derivations presented here are validated by a very good agreement between the numerical results of exact solutions and the results of elastic–plastic finite-element analysis by ABAQUS.

1. Introduction

Spherical pressure vessels are popular in engineering applications. Their structure design and safety assessment are issues of paramount importance. Limit analysis is efficient for structural design and safety assessment based on the static or the kinematic theorem [1]. In particular, it is possible to deal with limit analysis problems involving isotropic hardening effects and large deformation by sequential limit analysis [2]. By sequential limit analysis, we conduct a sequence of limit analysis problems by updating the yield function and the deformed configuration sequentially [2–18]. Further, a generalized Hölder inequality [19] has been utilized to establish the kinematic formulation of sequential limit analysis from the corresponding static formulation [2,5–18].

In literature, some attention has been paid to elastic–plastic spherical vessels made of isotropic hardening [e.g. Refs. [20–22]]. Actually, it is noted that real-life materials generally demonstrate a combined isotropic/kinematic hardening behavior [23]. Recently, Chaaba [24] originally investigated the upper-bound limit pressure of thick vessels of combined isotropic/kinematic hardening by sequential limit analysis based on the bipotential concept. Accordingly, we attempt to extend the approach of sequential limit analysis to consider spherical vessels made of combined isotropic/kinematic hardening materials by a generalized Hölder inequality [19]. Based on the previous work on isotropic hardening pressurized vessels [12,17], it is to utilize a generalized Hölder inequality [19] to establish the kinematic formulation from the corresponding static formulation of a sequential limit analysis problem. In the paper, the Armstrong-Frederick kinematic hardening model [25,26] is adopted and the Voce hardening law [27] is incorporated for isotropic hardening behavior. Both static and kinematic limit analysis are to be analytically conducted sequentially to approach the real limit solutions. Accordingly, exact solution is acquired by confirming the equality relation between the greatest lower bound and the least upper bound [e.g. Refs. [17,18]]. On the other hand, the onset of instability is also investigated to capture the occurrence of weakening phenomenon resulting from pressurized deformation [17]. In addition, elastic–plastic analysis is performed by the commercial finite-element code ABAQUS [28] incorporated with the user subroutine (UMAT) for comparisons and validations. Note that, we implement the material constitutive model of combined the Voce isotropic hardening [27] and the Armstrong-Frederick kinematic hardening [25,26] in the user subroutine UMAT for the commercial finite-element code ABAQUS [28].

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2. Analytical background

We consider thick-walled spherical vessels made of materials with nonlinear isotropic and kinematic hardening subjected to internal pressure in spherically symmetric conditions. It is assumed that the behavior of nonlinear isotropic and kinematic hardening is described by the Voce hardening law [27] and the Armstrong–Frederick kinematic hardening model [25,26], respectively. Corresponding to the nonlinear combined isotropic/kinematic hardening for a von Mises material, the yield function is denoted as [29].

$$f(\sigma - X) = \sqrt{\frac{3}{2}\left(S - X^{\text{dev}}\right) : \left(S - X^{\text{dev}}\right)} - \sigma_Y \quad (1)$$

where S is the deviatoric stress tensor, X^{dev} is the deviatoric part of the backstress tensor X acting to translate the center of the yield surface, σ_Y is the yield strength. It is noted that the backstress X denotes the movement of the yield surface center while the yield strength σ_Y accounts for the size of the yield surface. Accordingly, the convexity of the yield surface preserves for a von Mises material with nonlinear combined isotropic/kinematic hardening.

By the Armstrong–Frederick kinematic hardening model [25,26], the backstress rate $\dot{X}$ is described as

$$\dot{X} = \frac{2}{3}C\dot{\varepsilon} - \gamma X\dot{\bar{\varepsilon}} \quad (2)$$

where C and γ are material parameters, $\dot{\varepsilon}$ is the plastic strain rate, $\dot{\bar{\varepsilon}}$ denotes the equivalent plastic strain rate.

By the Voce hardening law [27], the behavior of nonlinear isotropic hardening is modeled as

$$\sigma_Y = \sigma_\infty - (\sigma_\infty - \sigma_0)\exp(-h\bar{\varepsilon}) \quad (3)$$

where σ_0 is the initial yield strength, σ_∞ is the saturation value of σ_0, h is the hardening exponent and $\bar{\varepsilon}$ is the equivalent plastic strain.

Moreover, we have the equivalent stress $\bar{\sigma}$ and the equivalent plastic strain rate $\dot{\bar{\varepsilon}}$ associated with the von Mises yield criterion expressed as follows

$$\bar{\sigma} = \sqrt{\frac{1}{2}\left[(\sigma_\theta - X_\theta) - \left(\sigma_\phi - X_\phi\right)\right]^2 + \frac{1}{2}\left[\left(\sigma_\phi - X_\phi\right) - (\sigma_r - X_r)\right]^2 + \frac{1}{2}\left[(\sigma_r - X_r) - (\sigma_\theta - X_\theta)\right]^2} \quad (4)$$

$$\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{9}\left[\left(\dot{\varepsilon}_\theta - \dot{\varepsilon}_\phi\right)^2 + \left(\dot{\varepsilon}_\phi - \dot{\varepsilon}_r\right)^2 + \left(\dot{\varepsilon}_r - \dot{\varepsilon}_\theta\right)^2\right]} \quad (5)$$

where σ_r, σ_θ and σ_ϕ are the stress components in the radial, polar and azimuthal directions, respectively. X_r, X_θ andX_ϕ are the backstress components in the radial, polar and azimuthal directions, respectively. $\dot{\varepsilon}_r$, $\dot{\varepsilon}_\theta$ and $\dot{\varepsilon}_\phi$ are the plastic strain rates components in the radial, polar and azimuthal directions, respectively.

Considering spherical symmetry and the incompressibility, the expressions of the equivalent stress $\bar{\sigma}$ and the equivalent plastic strain rate $\dot{\bar{\varepsilon}}$ can be simplified into the following forms

$$\bar{\sigma} = [-(\sigma_r - X_r) + (\sigma_\theta - X_\theta)] \quad (6)$$

$$\dot{\bar{\varepsilon}} = -\dot{\varepsilon}_r = 2\dot{\varepsilon}_\theta \quad (7)$$

Due to the spherical symmetry, we have the plastic strain rate–velocity relations as

$$\dot{\varepsilon}_r = \frac{du_r}{dr} \quad (8)$$

$$\dot{\varepsilon}_\theta = \dot{\varepsilon}_\phi = \frac{u_r}{r} \quad (9)$$

where u_r is the velocity component in the radial direction.

Considering the incompressibility, we obtain the radial velocity as

$$u_r = \frac{a^2\dot{a}}{r^2} \quad (10)$$

where a, $\dot{a}$ are the current interior radius and its velocity, respectively.

Thus, the equivalent plastic strain rate $\dot{\bar{\varepsilon}}$ can be expressed in the form as

$$\dot{\bar{\varepsilon}} = 2\frac{a^2\dot{a}}{r^3} \quad (11)$$

The equivalent plastic strain is then obtained as

$$\bar{\varepsilon} = \int \dot{\bar{\varepsilon}}dt = \ln\frac{r^2}{r_0^2} \quad (12)$$

where r_0 is the initial radius of the location concerned.

Combining Eqs. (2) and (11), the components of the backstress rate $\dot{X}$ can be described as follows

$$\dot{X}_r = \frac{2}{3}C\dot{\varepsilon}_r - \gamma X_r\dot{\bar{\varepsilon}} \quad (13)$$

$$\dot{X}_\theta = \frac{2}{3}C\dot{\varepsilon}_\theta - \gamma X_\theta\dot{\bar{\varepsilon}} \quad (14)$$

Due to the proportional loading and the initial condition $X(0) = 0$, the integral form of the Armstrong–Frederick kinematic hardening model [25,26] can be obtained as follows

$$X_r = -\frac{2C}{3\gamma} + \frac{2C}{3\gamma}e^{\gamma\varepsilon_r} = -\frac{2C}{3\gamma} + \frac{2C}{3\gamma}\left(\frac{r_0}{r}\right)^{2\gamma} \quad (15)$$

$$X_\theta = \frac{C}{3\gamma} - \frac{C}{3\gamma}e^{-2\gamma\varepsilon_\theta} = \frac{C}{3\gamma} - \frac{C}{3\gamma}\left(\frac{r_0}{r}\right)^{2\gamma} \quad (16)$$

Combining Eqs. (3) and (12), we express the Voce hardening law [27] in the form as

$$\sigma_Y = \sigma_\infty - (\sigma_\infty - \sigma_0)\left(\frac{r_0}{r}\right)^{2h} \quad (17)$$

Accordingly, we have acquired the values of the backstress X and the yield strength σ_Y expressed as functions of the configuration. Namely, we can update the yield function with these step-wise