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Thermoelastic creep analysis of a functionally graded various thickness rotating disk with temperature-dependent material properties

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ABSTRACT

A semi-analytical solution for rotating axisymmetric disks made of functionally graded materials was previously proposed by Hosseini Kordkheili and Naghdabadi [1]. In the present work the solution is employed to study thermoelastic creep behavior of the functionally graded rotating disks with variable thickness in to the time domain. The rate type governing differential equations for the considered structure are derived and analytically solved in terms of rate of strain as a reduced to a set of linear algebraic equations. The advantage of this method is to avoid simplifications and restrictions which are normally associated with other creep solution techniques in the literature. The thermal and structural properties of the base metal are also considered as a function of temperature. It is noted that ignoring the temperature dependency of these properties caused up to 200% errors in the creep solution results. Also, results for the strain rates presented due to centrifugal force and thermal loadings for different disk cross section profiles as well as different boundary conditions. Results obtained within this solution are verified with those available in the literature for easier cases.

1. Introduction

Utilizing high power gas turbine in various industries is rapidly increased. Gas turbines are employed in aerospace engineering to produce demanded trust for huge aircraft and spacecraft vehicles. However in oil and gas sector industries gas turbines are providing requested head for running pure gas and oil and/or their products into the national or international lines. The ground power gas turbines are also used to produce electricity in power planets most time as reserve for peak demand. Turbine rotating disks, with variable thickness on radius, are the most important part of the gas turbine engine which is subjected with centrifugal force due to rotating velocity as well as high temperature gradient within operating environment. Due to long operational time, gas turbine rotating disks are susceptible to creep. Therefore the creep behavior has to be considered during the stress analysis of this structure.

Selecting proper material before carrying out the stress analysis, will lead to more well designed structure. During last two decades researchers proposed Functionally Graded Materials (FGMs) to be employed to fabricate structures that work in high thermal and mechanical loading conditions. In FGMs the material properties are varied continuously through the thickness according to a power-law distribution of volume fraction of the constituents. As the use of FGMs increases, new methodologies have to be developed to characterize FGMs, and also to analyze and design structural components made of these materials. There have been some studies on thermal and mechanical creep analysis in the basic structural components of FGMs. Yang [2] presented an analytical solution for the calculations of time-dependent stresses in FGM for the elastic and creep behavior of materials. He obtained Naviour differential equation for FGM. But to simplify the solution he approximated radially varying elastic modulus by and $\alpha T = Br^m$, where r is the radius direction parameter, α is thermal expansion coefficient and T is through-the-radius temperature distribution. Singh and Ray [3] investigated creep behavior of a rotating disk made of isotropic FGM. They assumed a linear distribution for volume fraction. The disk that they considered also had a constant thickness and was subjected with only mechanical loading due to angular velocity. They also assumed Elastic deformations are small and neglectable as compared to creep deformation. Jahed and Bidabadi [4] presented a general axisymmetric method for an inhomogeneous body for a disk with varying thickness. An approximation has been employed during their solution algorithm.

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It means that they avoid considering the differentiation constitutive terms of governing equations for creep analysis. Gupta et al. [5] investigated creep behavior of a rotating FG disk made of silicon carbide particles in a matrix of pure aluminum by using Cherby's law. The FG disk that they considered had a constant thickness and assumed linear particles distribution. Also they assigned a fixed function for radius direction temperature distribution. You et al. [6] investigated steady state creep behavior of thick walled FG cylindrical vessels involving Norton's law. They used a simple and accurate method to determine creep stress and strain rates in thick-walled cylinders subjected by only internal pressure. Singh [7] studied creep in an FG rotating disk with constant thickness considering Norton's law. In order to stress analysis, by substituting Norton's law, into the generalized constitutive equations for creep, he first found strain rates relations. Using these relations, he then applied a direct integrating scheme to calculate nonlinear stress components relations. He finally employed an iterative numerical method to find stress distribution in disk radius direction. He also avoids considering elastic deformation. To get rapid convergence during iterative solution, he also considered a linear relation between current and previous stress values during iterative solution. Pankaj and Bansal [8] obtained creep stresses and strain rates for a thin rotating disk with variable density. They used seth's generalized principal strain measure as well as seth's transition theory to investigate elastic–plastic transition. They observed that the acceptable angular speed of disk depends on the directional increasing density through the radius. Bayat et al. [9] used the Hosseini Kordkheili and Naghdabadi's [1] semi-analytical scheme to obtain mechanical and thermal stresses in FG rotating disk with variable thickness. Bayat et al. [10] also presented a thermoelastic analysis for axisymmetric rotating disks made of FGMs with variable thickness. They concluded that for a given pair of materials there is a particular volume fraction that gives maximum specific mechanical response under thermal loading. Deepak et al. [11] investigated creep behavior of rotating discs made of FGMs with linearly varying thickness. For a given pair of materials they showed that there is a particular volume fraction that gives maximum specific mechanical response under thermal loading.

This work aims to propose a solution for the most general Naviour thermoelastic creep equation of a rotating FG disk (Fig. 1) with variable thickness and temperature-dependent properties in which is subjected by through-the-radius varying thermal and mechanical loadings. For this purpose, the equilibrium equations are derived based on the thermoelasticity theory. The radial domain of the disk is divided into a number of finite sub-domains, in which the thermomechanical properties are assumed to be constant within each sub-domain. Imposing the continuity conditions at the interface of the adjacent sub-domains, together with the global boundary conditions, a set of linear algebraic equations are derived. Solving the linear algebraic equations, the thermoelastic responses for a hollow or solid rotating axisymmetric FG disk are obtained. The relative influence of basic factors, such as property gradation, centrifugal body loading and thermal loading for different thickness profiles as well as different boundary conditions are investigated.

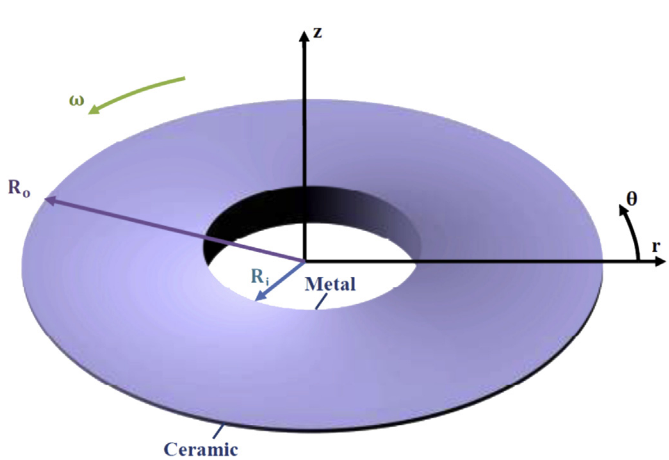

Fig. 1. FG rotating disk with various thickness.

2. Governing equations

Consider the axisymmetric FG rotating disk at an angular velocity ω, Fig. 1, with various thickness $h_{(r)}$ and inner and outer radiuses R_i and R_o, respectively. Four different types of profiles for variation of the thickness in radius direction are also considered in this study as Fig. 2 [9,10].

2.1. Gradiation relation

The FG rotating disk is assumed to be made of a mixture of two constituent materials. The inner and outer layers of the disk are metal-rich and ceramic-rich surfaces, respectively. There are various models for expressing the variation of material properties between these two surfaces in the literature. The most commonly used of these models is the power law distribution of the volume fraction. i.e.

$$P_{(r,T)} = \left(P_{o_{(T)}} - P_{i_{(T)}}\right)\left(\frac{r-R_i}{R_o-R_i}\right)^n + P_{i(T)} \quad ; \quad R_i < r < R_o \tag{1}$$

where subscripts i and o denote the metal and the ceramic properties, respectively. n is the so-called volume fraction index also r and T stand for radius and temperature, respectively. Fig. 3 shows the distribution of the metal volume fraction through the disk radius with $R_o = 5R_i$ for various values of the grading index n. This study assumes that the elastic modulus, E, mass density, ρ, the Poisson's ratio, v, thermal expansion coefficient, α, and heat conductivity, k, vary through the disk radius according to the gradation relation (1). According to Reddy' report [12] temperature dependent material properties are obtained using the following relation

$$p_{(T)} = p_0\left(\frac{c_{-1}}{T} + 1 + c_1T + c_2T^2 + c_3T^3\right) \tag{2}$$

where p_0, c_{-1}, c_1, c_2 and c_3 are constants in the cubic fit of the material.

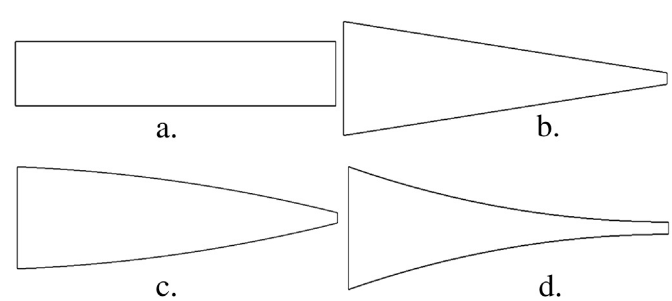

Fig. 2. Four different types of profiles cross section of rotating disc. a. Constant, b. Linear, c. Convex and d. Concave.