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Solutions of the second elastic–plastic fracture mechanics parameter in test specimens under biaxial loading

Ping Ding, Xin Wang*

Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa, Ontario K1S 5B6, Canada

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ABSTRACT

Extensive finite elements analyses have been conducted to obtain solutions of the A-term, which is the second parameter in a three-term elastic–plastic asymptotic expansion, for test specimens under biaxial loading. Three mode I plane-strain test specimens, *i.e.* single edge cracked plate (SECP), center cracked plate (CCP) and double edge cracked plate (DECP) were studied. The crack geometries analyzed include shallow to deep cracks, and the biaxial loading ratios analyzed are 0.5 and 1.0. Solutions of A-term were obtained for materials following the Ramberg–Osgood power law with hardening exponent of $n = 3$, 4, 5, 7 and 10. Remote tension loading was applied which covers from small-scale to large-scale yielding. Based on the finite element results, effects of biaxial loading on crack tip constraint were discussed. Empirical equations to predict the A-term under small-scale yielding to fully-plastic condition were developed using estimation methods developed earlier. Based on the relationships between A and other commonly-used second fracture parameter Q and A_2, the present solutions can be used to calculate parameters Q and A_2 as well. The results presented in the paper are suitable to determine the second elastic–plastic fracture parameters for test specimens for a wide range of crack geometries, material strain hardening behaviors under biaxial loading conditions.

1. Introduction

In classical elastic–plastic fracture mechanics (EPFM), one-parameter approach, which describes the HRR fields [1,2] based on J-integral [3], usually can work well for high constraint cases. For low constraint cases, under high loading conditions, the dominance of J-integral will be lost, and the one-parameter approach of J-integral will not be appropriate any more.

Two-parameter approaches have been developed to overcome the limitation of the EPFM one-parameter approach, in which a second fracture mechanics parameter is introduced to characterize the constraint effect besides the load-related parameter J-integral. Several commonly-used two-parameter approaches are, J–T [4–6], J–Q [7,8] and J–A_2 (or J–A) [9–12] approaches, where constraint parameter A is a different normalizing form of A_2 [11,12].

Determination of both J-integral and second fracture mechanics parameter, T, Q or A_2 (A), is the precondition of application of J–T, J–Q and J–A_2 (A) approaches. In the early development of EPFM, J-integral solutions have been well established. The solutions of constraint parameter T-stress have also been well established in the literature. Currently, numerical method, such as the finite element analysis (FEA) method, is the main method for the determination of constraint parameters Q and A_2 (A). For example, Nikishkov et al. [12] suggested an algorithm which determines solutions of A using least squares procedure based on the finite element analysis results. Although with high accuracy, numerical method is a time-consuming way to obtain solutions of parameters. Consequently, understanding of constraint effect near crack-tip is limited because of the scarcity of solutions of constraint parameter A_2 (A) and Q. Only one estimation method for parameter Q determination, which was suggested by O'Dowd [13], can be found in the literature. No approximation method for predicting A_2 or A is available until recently. For the convenience of engineering and theoretical applications, systematic development of estimation methods for A_2 (A) and Q is required.

In recent works of authors [14,15], three estimation methods for determining constraint parameter A conveniently have been developed, namely, curve shape similarity method, T-stress-based method and fully plastic analysis (superposition) method. In Ref. [14], the relationships between A and other two commonly-used constraint parameters A_2 and Q were presented, by which solutions of A_2 and Q can be determined directly from obtained A

* Corresponding author. Tel.: +1 613 520 2600x8308; fax: +1 613 520 5715.
E-mail address: xwang@mae.carleton.ca (X. Wang).

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solutions. We have successfully applied the three estimation methods of A to 2D plane strain cracked specimens under uniaxial loading conditions, see Refs. [14,15]. Biaxial loading is experienced by many engineering components, such as those in pressure vessels and piping structures. They are thus of equal theoretical and engineering practical significance as the uniaxial loading cases. In the present work, the three estimation methods developed in Refs. [14,15] will be applied to develop A solutions for biaxial loading cases for three mode I crack plane-strain specimens, single edge cracked plate (SECP), center cracked plate (CCP) and double edge cracked plate (DECP).

The rest of the paper is organized as follows. In section 2, theoretical background for J–A approach, and the estimation methods for parameter A will be summarized. Extensive finite element analyses will be presented in section 3 for SECP, CCP and DECP cracked specimens under biaxial loading conditions with biaxial ratios $\lambda = 0.5, 1.0$, to determine numerical solutions of constraint parameter A, through the least squares fitting method suggested by Nikishkov et al. [12]. Using the obtained numerical solutions of A, constraint effect near crack-tip of specimens will be discussed. It will be demonstrated that biaxial loading has a significant effect on the constraint parameters. In section 4, based on the obtained numerical solutions of A, the three estimation methods for A will be applied to the SECP, CCP and DECP specimens under biaxial loading conditions. Approximation formulas for estimation of A corresponding to each of the three methods will be developed, and values of coefficients in those approximation formulas will be determined and tabulated. Conclusions will be drawn in Section 5. The results in this work will show that, generally, all three estimation methods for constraint parameter A can be applied effectively to biaxial loading conditions of cracked specimens.

2. Theoretical background

2.1. J–A approach and numerical determining of constraint parameter

As mentioned in the introduction section, J–A two-parameter approach of elastic–plastic fracture mechanics, which is suggested by Nikishkov et al. [11,12], is an alternate format of J–A_2 approach developed by Yang et al. [9,10]. In these two formats of the approaches, the constraint parameter A or A_2 represents the magnitude of the second term in a three-term series expansion of crack-tip stress fields. In this paper, the J–A format is used.

Considering a two-dimensional elastic–plastic specimen containing a mode I crack under plane strain condition, if the elastic–plastic behavior of the specimen material described by deformation theory follows the Ramberg–Osgood relationship, the uniaxial stress–strain curve can be described as:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n \tag{1}$$

where α is a material coefficient, n is the hardening exponent ($n > 1$), $\varepsilon_0 = \sigma_0/E$, E is Young's modulus, σ is the stress applied on remote end of specimen, and σ_0 is the material yield stress.

In the J–A two-parameter approach suggested by Nikishkov et al. [11,12], for the hardening exponent $n \geq 3$ and under plane-strain conditions, the three-term asymptotic solution expression for stress field near the crack-tip in an elastic–plastic material is given as:

$$\frac{\sigma_{ij}}{\sigma_0} = A_0\bar{r}^s\bar{\sigma}_{ij}^{(0)}(\theta) - A\bar{r}^t\bar{\sigma}_{ij}^{(1)}(\theta) + \frac{A^2}{A_0}\bar{r}^{2t-s}\bar{\sigma}_{ij}^{(2)}(\theta) \tag{2}$$

In Eq. (2), $\sigma_{ij}(\bar{r}, \theta)$ are stress components, σ_r, σ_θ and $\sigma_{r\theta}$ in a polar coordinate system with origin at the crack tip $\bar{\sigma}_{ij}^{(0)}(\theta)$, $\bar{\sigma}_{ij}^{(1)}(\theta)$ and $\bar{\sigma}_{ij}^{(2)}(\theta)$ are normalized angular stress functions. The dimensionless radius $\bar{r}$ is defined as $\bar{r} = r/(J/\sigma_0)$, where J is the J-integral at the crack tip. Power t is an eigenvalue depending on the hardening exponent n of Ramberg–Osgood relation, and power $s = -1/(n+1)$. The polynomial coefficient A_0 is defined as [11], $A_0 = (\alpha\varepsilon_0 I_n)^{-1/(n+1)}$, where I_n is a scaling integral only depending on n, see Refs. [1,2]. Nikishkov [11] has proposed a computational algorithm to determine the values of normalized angular functions $\bar{\sigma}_{ij}^{(0)}(\theta)$, $\bar{\sigma}_{ij}^{(1)}(\theta)$ and $\bar{\sigma}_{ij}^{(2)}(\theta)$, asymptotic power t, and scaling integral I_n. The three-term expansion in Eq. (2) for the crack tip stress (displacement) fields is controlled by two parameters, the magnitude of the first term (J-integral) and a second parameter (A) controlling the second and third term. Functions $\bar{\sigma}_{ij}^{(0)}(\theta)$, $\bar{\sigma}_{ij}^{(1)}(\theta)$ and $\bar{\sigma}_{ij}^{(2)}(\theta)$, asymptotic power t, and scaling integral I_n for different n values can be found from Refs. [11] and [16]. It is worth noting, as shown in Refs. [14] and [15], that the present A parameter [12] and the original A_2 parameter by Chao and co-workers [9] and [10], are related by:

$$A = -(\alpha\varepsilon_0 I_n)^s\left(\frac{J}{\sigma_0 L}\right)^{t-s}A_2 \tag{3}$$

where L is a characteristic length parameter. It has been well established in Refs. [9] and [10] that A_2 will be independent of load under fully plastic loading conditions. However, parameter A will vary with load through J, as shown in Eq. (3).

The application of J–A (A_2) two-parameter approach depends on the determination of the load-related parameter J and constraint parameter A (A_2). The solutions of J-integral (including analytical, numerical and approximate solutions) have been well established in the literature, such as the numerical solution of J suggested by Moran and Shih [17], which has been adopted in the commercial code ABAQUS [18] utilized in the present research.

Based on some stress component with the corresponding finite element solutions at one or several locations (points) within the plastic zone, Yang et al. [9,10] determine the A_2 values by matching the three-term expansion on crack tip stress field. It is called the "*point match*" method by some researchers. Also based on FEA results, Nikishkov et al. [12] suggested a "*fitting method*" to determine parameter A values by Eq. (2), which is capable of obtaining numerical value of A more accurately. In the present work, the fitting method proposed in Ref. [12] is utilized to obtained numerical solutions of constraint parameter A, which are used as the accurate solutions of A in this investigation. See Refs. [12] and [14] for more details about the procedure of the fitting method.

Determining the constraint parameter A (A_2) numerically is a time-consuming work, therefore the authors [14,15] have developed three estimation methods to obtain the values of parameter A conveniently. The three approximation methods will be reviewed in the next section.

2.2. Estimation methods of constraint parameter A

2.2.1. Estimating A solutions by curve shape similarity

In Ref. [14], an estimation method (curve shape similarity method) has been proposed to approximate parameter A solutions for any hardening exponent n based on A solutions for a specific n value. The method was developed based on a phenomenon observed by Nikishkov et al. [12]. Based on FEA results for cracked specimens under uniaxial loading, they found that, the curves of parameter A vs. external loading ratios σ/σ_L for different values of hardening exponent n have the similar shape and the curves only differ from each other by a constant which can be obtained from the