International Journal of Pressure Vessels and Piping 111-112 (2013) 321–330

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

On numerical analysis of composite and laminated cylinders of finite length subjected to partially distributed band load

Payal Desai [a,*], Tarun Kant [b]

[a] School of Engineering and Technology, Navrachana University, Vadodara, India
[b] Department of Civil Engineering, Indian Institute of Technology Bombay, Mumbai, India

ARTICLE INFO

Article history:
Received 3 January 2013
Received in revised form
18 September 2013
Accepted 7 October 2013

Keywords:
Semi analytical method
Circular cylinder
Band load
Laminates

ABSTRACT

A simplified and accurate semi analytical – numerical model is presented here to investigate the behavior of cylinders of finite length subjected to partially distributed band load. A diaphragm supported laminated cylinder under symmetric load which is considered as a two dimensional (2D) plane strain problem of elasticity in (r, z) direction. The boundary conditions are satisfied exactly in axial direction (z) by taking an analytical expression in terms of Fourier series expansion. Fundamental (basic) dependent variables are chosen in the radial coordinate of the cylinder. First order simultaneous ordinary differential equations are obtained as mathematical model which are integrated through an effective numerical integration technique by first transforming the boundary value problem (BVP) into a set of initial value problems (IVPs). The proposed method is successful in handling the 2D and three dimensional (3D) elasticity problems involving wide range of loadings, material properties and mixed variables. For cylinders subjected to band load, the convergence study is carried out and presented for different harmonics. The numerical results obtained are also first validated with existing literature for their accuracy.

1. Introduction

A circular cylinder is such a structural element which is used extensively. Therefore, calculation of accurate state of stress in cylinder under various loading conditions is of considerable attention. The most studied stressed state is cylinder loaded with uniformly distributed pressure over it length. There is much less information for determination of state of stress in a zone of a circular area of its length. This makes the problem challenging to analyze. Before these cylinder devices are used in engineering design, it is very important that these are analyzed very accurately. For such a reason, present study focuses the analysis of composite and laminated cylinders subjected to band load using the simple yet accurate semi analytical cum numerical methodology. The uniqueness of this approach is: it first requires algebraic manipulation of basic elasticity equations like equilibrium, strain displacement and constitute equations. After this manipulation, this becomes the two point boundary value problem (BVP) which governs the behavior of finite length cylinder which is plane strain two dimensional problem in r, z plane and gives four first order simultaneous partial differential equations. Taking a clue from this development, an attempt is made here to extend the strategy of transforming the governing system of PDEs to a system of ODEs [1] for elastostatic problems; Preliminary involvement of such formulation for a class of classical shell problems is seen in the literature [2,3].

Recently [4,5], have obtained accurate stresses in laminated finite length cylinders subjected to thermo elastic uniformly distributed and sinusoidal load using similar numerical model. Also, with the same methodology, accurate numerical results for composite and sandwich narrow beams were obtained by Kant et al. [6]. Some of the literature pertaining to problem of a cylinder is described as follows. The classic problem of an infinitely long elastic cylinder of an isotropic material under internal and external pressure was analyzed first by Lame in 1847 given in Ref. [7] and by Lekhnitskii [8] for anisotropic and layered materials in his book. This particular problem has been studied by many during later years. Ref. [9] obtained stresses and displacements by the use of 3D elasticity theory and several shell theories in a long isotropic circular cylinder subjected to an axisymmetric radial line load externally and compared results with the shell theories of Love and Flugge. An elasticity solution by using a Love function approach for semi-infinite circular cylindrical shell subjected to a concentrated axisymmetric radial line load at the free end were presented in Ref.

* Corresponding author.
E-mail addresses: payaldesai79@gmail.com, payald@nuv.ac.in (P. Desai).

0308-0161/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijpvp.2013.10.001

[10]. The problem of an infinite circular cylindrical shell subjected to periodically spaced band loads using 3D elasticity theory and the shell theories of Love (and Donnell), Flugge, and a theory developed by Reissner and by Nagdhi was solved by Klosner et al. [11]. An approximate solution to the Navier equations of the 3D elasticity for an axisymmetric orthotropic infinitely long circular cylinder subjected to internal and external pressure, axial loads, and closely spaced periodic radial loads are obtained by Misovec et al. [12]. Clamped–clamped and clamped–simply supported cylindrical shells by a so-called segmentation numerical integration technique were analyzed in Ref. [13]. The same technique for elastic analysis of cylindrical pressure vessels with various end closures using classical theory for thin axisymmetric shells of revolution is used in Ref. [14]. An axisymmetric stress analysis of a transversely isotropic, short hollow cylinder subjected to an outer band load presented in a series form was given in Ref. [15]. The generalized Elliott's solution is used for the analysis. The solution consists of five independent potential functions which yield two kinds of elasticity solution. The boundary conditions for the shearing stress on the four surfaces are exactly satisfied. Other boundary conditions are numerically satisfied with the aid of a Fourier series expansion or a Fourier–Bessel series expansion. The stresses in thick-wall cylinders under non-uniform external pressure by means of Airy's stress function were determined in Ref. [16]. A state space approach is commonly in use among the researchers in recent years. Axisymmetric deformation of laminated hollow cylinders with simply supported and clamped edges, based on the basic equations of elasticity obtained by Sheng et al. [17].

In this paper, governing anisotropic elasticity equations of a simply (diaphragm) supported laminated cylinder are used to predict its behavior subjected to partially distributed band load. By assuming a global analytical solution in the longitudinal direction satisfying the two end boundary conditions exactly. The equations are reformulated to enable application of an efficient and accurate numerical integration technique for the solution of the BVP of a cylinder in the radial coordinate. To enable application of numerical integration, BVP of a cylinder is converted into a set of IVPs. The basic approach to convert a BVP into a set of IVPs is also explained in the following sections. Numerical results are validated with those given in literature. Present formulation enables incorporation of any number of layers in case of laminated cylinders and various loading conditions.

2. Problem formulation

The behavior of a cylinder is mathematically formulated as two point BVP governed by a set of linear first order ODEs. This can be written mathematically by the following equation [2,3].

$$\frac{\mathrm{d}}{\mathrm{d}r}y(r) = A(r)y(r) + p(r) \quad (1)$$

In the domain, $r_1 \le r \le r_2$, where, $y(r)$ is an n-dimensional vector of dependent variables; dependent variables in the present case can be described as $\underline{y} = (u, w, \sigma_r, \tau_{rz})^t$. Choice of dependent variables is an important task. The variables which naturally appear on $r =$ constant are chosen as dependent variables; such variables are called intrinsic variables. Remaining variables are described as auxiliary dependent variables which are dependent on intrinsic dependent variables. $A(r)$ is a coefficient matrix of ordinary differential equations. $p(r)$ is an n-dimensional vector of non-homogeneous (loading) terms. For boundary conditions, any $n/2$ elements of $y(r)$ are specified at the two termini edges; mixed type of boundary conditions can be specified in this type of formulation.

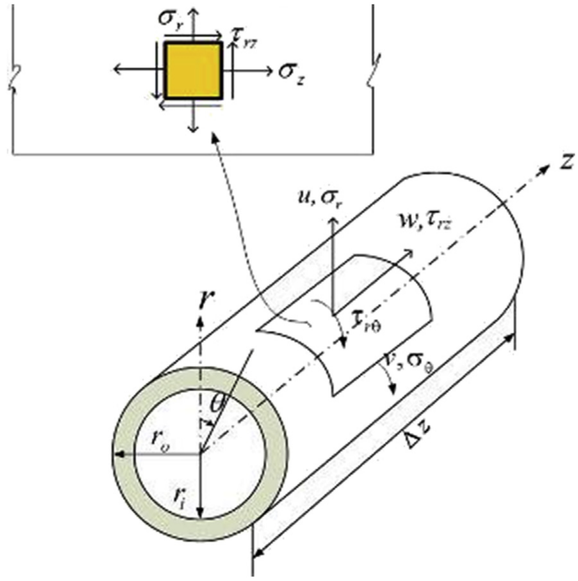

Fig. 1. Coordinate system and geometry of cylinder.

2D stress equilibrium equations for finite length cylinder in cylindrical coordinates can be written as (Fig. 1),

$$\sigma_{r,r} + \tau_{rz,z} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \tau_{rz,r} + \sigma_{z,z} + \frac{\tau_{rz}}{r} = 0 \quad (2a)$$

2D strain–displacement and in cylindrical coordinates are

$$\varepsilon_r = u_{,r}; \varepsilon_\theta = \frac{u}{r}; \varepsilon_z = w_{,z}, \varepsilon_{rz} = u_{,z} + w_{,r} \quad (2b)$$

Stresses in terms of displacement components for cylindrically orthotropic material can be cast as follows:

$$\sigma_r = C_{11}u_{,r} + C_{12}\frac{u}{r} + C_{13}w_{,z}, \sigma_\theta = C_{21}u_{,r} + C_{22}\frac{u}{r} + C_{23}w_{,z},$$
$$\sigma_z = C_{31}u_{,r} + C_{32}\frac{u}{r} + C_{33}w_{,z}, \tau_{rz} = G(w_{,r} + u_{,z}) \quad (2c)$$

where,

$$\nu_{r\theta} = \frac{\nu_{\theta r}}{E_\theta}E_r, \ \nu_{rz} = \frac{\nu_{zr}}{E_z}E_r, \ \nu_{z\theta} = \frac{\nu_{\theta z}}{E_\theta}E_z$$
$$C_{11} = \frac{E_r(1-\nu_{\theta z}\nu_{z\theta})}{\Delta}, \ C_{12} = \frac{E_r(\nu_{\theta r}+\nu_{zr}\nu_{\theta z})}{\Delta}, \ C_{13} = \frac{E_r(\nu_{zr}+\nu_{\theta r}\nu_{z\theta})}{\Delta}$$
$$C_{22} = \frac{E_\theta(1-\nu_{rz}\nu_{zr})}{\Delta}, \ C_{32} = \frac{E_\theta(\nu_{z\theta}+\nu_{r\theta}\nu_{zr})}{\Delta} \ C_{33} = \frac{E_z(1-\nu_{r\theta}\nu_{\theta r})}{\Delta}$$
$$\text{where } \Delta = (1 - \nu_{r\theta}\nu_{\theta r} - \nu_{\theta z}\nu_{z\theta} - \nu_{zr}\nu_{rz} - 2\nu_{\theta r}\nu_{z\theta}\nu_{rz})$$
$$C_{21} = C_{12}, \ C_{23} = C_{32}, \ C_{31} = C_{13}$$

Boundary conditions in the longitudinal and radial directions are,

Table 1
Initial and integrated values.

N	Initial boundary				Terminal boundary				Load term
	u	w	σ_r	τ_{rz}	u	w	σ_r	τ_{rz}	
0	0	0	0(S)	0(S)	$Y_{1,0}$	$Y_{2,0}$	$Y_{3,0}$	$Y_{4,0}$	I
1	1	0	0	0	$Y_{1,1}$	$Y_{2,1}$	$Y_{3,1}$	$Y_{4,1}$	D
2	0	1	0	0	$Y_{1,2}$	$Y_{2,2}$	$Y_{3,2}$	$Y_{4,2}$	D
FI	X_1	X_2	0(S)	0(S)	C	C	C	C	I