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Analysis of residual stresses in a long hollow cylinder

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ABSTRACT

This paper presents an analytical method for solving the axisymmetric stress problem for a long hollow cylinder subjected to locally-distributed residual (incompatible) strains. This method is based on direct integration of the equilibrium and compatibility equations, which thereby have been reduced to the set of two governing equations for two key functions with corresponding boundary and integral conditions. The governing equations were solved by making use of the Fourier integral transformation. Application of the method is illustrated with an analysis of the welding residual stresses in a butt-welded thick-walled pipe.

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1. Introduction

Welding residual strains substantially affect the lifetime of welded structural members [1]. The locally-distributed fields of technological residual strains occur in welded joints due to non-uniform heating and cooling during the welding and aftercooling processes as a result of phase change in the zones of thermal influence ([2], [3]: pp 75–87). Under certain conditions, the residual stresses cause brittle failure of long-term-functioning welded structures. Since an analysis of the residual stress-state induced by the welding residual strains is very important for a proper inspection of welded joints, it presents a vital issue for specialists in both academia and industry.

There are different approaches for analysis of the residual stresses [3,4] in butt-welded joints, which mostly combine both experimental [5–8] and theoretical [9,10] techniques. A vast majority of the existing experiment-calculated methods employ numerical [11–18] or approximate [19,20] procedures. However, for efficient analysis of the residual stresses, as well as for solving inverse elasto-plastic problems (which necessarily occur when one applies certain experiment-calculated methods), exact analytical solutions are required.

One of the most efficient theoretical approaches to determination of the residual stresses is based on the method of conventional plastic strains [19,21]. According to this method, the material of a welded solid is assumed to be elastic at some distance from the axes of welded joints. But within the zones of thermal influence due to welding, the material is assumed to be elastic-plastic. Consequently, the components of the strain-tensor can be presented in the form

$$e_i = \tilde{e}_i + \epsilon_i, \quad e_{ij} = \tilde{e}_{ij} + \gamma_{ij}, \ \ ij = ji, \ \ i \neq j, \tag{1}$$

where the indexes i and j show the coordinate directions in the chosen coordinate system; e_i and e_{ij} are the normal and shear strains-tensor components, respectively; $\tilde{e}_i$ and $\tilde{e}_{ij}$ denote the elastic strains; ϵ_i and γ_{ij} stand for the residual strains. The residual strains are locally-distributed in the neighborhood of a welded joint and vanish at a distance from it. In more general form, the representation of total strains has been given in [9,22]. The method of conventional plastic strains has been sufficiently employed for analysis of the residual stresses in welded structure members of various shape [23–25].

Among structural elements of different shape, cylindrical bodies are widely used in engineering practice as elements of pipelines, pylons, crosstops, pressure vessels, etc. In most cases, the residual stresses in butt-welded hollow cylinders can be analyzed with application of an axisymmetric model. By making use of the aforementioned method of conventional plastic strains, the problem for determination of the residual stresses in a butt-welded cylindrical pipe can be reduced to the axisymmetric elasto-plastic problem and then solved by means of the methods of elasticity.

There exist a great number of methods for determination of the stresses and displacements in a long hollow cylinder subjected to various types of external loading. A great many of the dominant methods are based on application of harmonic or biharmonic stress

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functions or displacement potentials ([26]: pp. 246–256, [27]: pp. 376–416, [28]: p. 123, [29]: pp. 343–397, [30]: pp.54–62). One of the earliest approaches has been based on application of two coupled harmonic displacement potentials, also known as Helmholtz functions [31]. Love [32] represented a general solution to the axisymmetric problem in terms of one biharmonic function. Different dominant methods are based on solutions in the form suggested by Weber [33], Timpe [34], Papkovich-Neuber [35,36], and Michell [37]. The solution of the non-axisymmetric problems is more complicated. Complete three-dimensional solution for a long elastic cylinder has been given by Dougal [38]. Having employed one of the aforementioned methods, the stresses or displacements can be represented in terms of potential functions. Hence, the solution of the axisymmetric problem is reduced to the boundary value problem for the corresponding harmonic or biharmonic functions. However, a proper construction of such functions presents a real challenge to engineering mathematics.

An alternative method to treat the axisymmetric elasticity and thermoelasticity problem for a long cylinder subjected to external force loading on its lateral surface or distributed temperature field was recently given in [39,40]. This method substantially uses the relations between the stress-tensor components obtained by direct integration of the equilibrium equations. By making use of such relations, the compatibility equations are reduced to the governing equations for one or two stress-tensor components, which were chosen to be the key functions. In a similar manner, the set of boundary conditions for different stress-tensor components is reduced to the boundary and integral conditions for the key functions. The obtained problems for the key functions are solved by means of the integral transformation method. After the key functions are found, the searched-for stress-tensor components can be found from the aforementioned relations between different stresses. This solution strategy allows one to construct an exact analytical solution without unjustified increasing degree of the governing differential equations. This direct integration method has also been employed for analysis of the welding residual stresses in an infinite layer [41–43] and butt-welded rectangular plates [44].

In this paper, we extend the direct integration method for analysis of the stresses in a long hollow cylinder exposed to the field of axisymmetric residual strains. Numerical computation is demonstrated with an example of welding residual stresses in a thick-walled butt-welded pipe.

2. Formulation of the problem

Consider a long hollow cylinder of inner and outer radii R_i and R_o, respectively. In the dimensionless cylindrical-polar coordinate system (r,φ,z), the cylinder occupies a domain $k \le r \le 1$, $0 \le \varphi \le 2\pi$, $-\infty < z < \infty$, where $r = R/R_o$, $k = R_i/R_o$, and $z = Z/R_o$, R and Z are the dimensional radial and axial coordinates, respectively. In the absence of body forces and external force loadings, the cylinder is stressed by the field of locally distributed and axisymmetric residual strains. The stress–strain problem for the cylinder is governed ([30], p.54) by the equilibrium equations

$$\frac{\partial}{\partial r}(r\sigma_r) + r\frac{\partial \tau_{rz}}{\partial z} = \sigma_\varphi, \tag{2}$$

$$\frac{\partial}{\partial r}(r\tau_{rz}) + r\frac{\partial \sigma_z}{\partial z} = 0 \tag{3}$$

in terms of stresses and the compatibility (continuity) equations

$$r\frac{\partial e_\varphi}{\partial r} = e_r - e_\varphi, \tag{4}$$

$$r\frac{\partial^2 e_\varphi}{\partial z^2} - \frac{\partial e_{rz}}{\partial z} + \frac{\partial e_z}{\partial r} = 0 \tag{5}$$

in terms of strains. Here σ_r, σ_φ, σ_z, and τ_{rz} denote the stress-tensor components. According to Eq. (1), the constitutive equations take the form:

$$\begin{aligned} Ee_r &= \sigma_r - \nu(\sigma_\varphi + \sigma_z) + E\epsilon_r, \\ Ee_\varphi &= \sigma_\varphi - \nu(\sigma_r + \sigma_z) + E\epsilon_\varphi, \\ Ee_z &= \sigma_z - \nu(\sigma_r + \sigma_\varphi) + E\epsilon_z, \\ Ge_{rz} &= \tau_{rz} + G\gamma_{rz}, \end{aligned} \tag{6}$$

where E stands for Young's modulus, $G = E/(2 + 2\nu)$ is the shear modulus, and ν denotes Poisson's ratio.

Due to the absence of external force loadings, the radial and shear stresses should meet the homogeneous boundary conditions

$$\sigma_r = 0, \quad r = \{k, 1\}, \tag{7}$$

$$\tau_{rz} = 0, \quad r = \{k, 1\} \tag{8}$$

on the inner and outer lateral surfaces of the cylinder. We also assume that the residual strains and, consequently, the corresponding stresses vanish, when $|z| \to \infty$.

In the formulated problem, the object is to determine analytical expressions for the stress-tensor components from Eqs. (2)–(6) due to the given residual strains ϵ_t $(t = \{r,\varphi,z\})$ and γ_{rz} under the homogeneous boundary conditions (7) and (8).

3. Construction of the solution

To find the stress-tensor components, we represent the compatibility equations (4) and (5) in terms of stresses and then reduce them to the governing equations for the key stresses. By making use of Eqs. (6) and (2), we can represent Eq. (4) in the form

$$r\frac{\partial}{\partial r}(\sigma_r + \sigma_\varphi) - \nu r\frac{\partial \sigma_z}{\partial r} + r(1+\nu)\frac{\partial \tau_{rz}}{\partial z} = E\left(\epsilon_r - \frac{\partial}{\partial r}(r\epsilon_\varphi)\right). \tag{9}$$

Putting the corresponding equations of (6) into Eq. (5) yields

$$\begin{aligned} &r\frac{\partial^2}{\partial z^2}(\sigma_\varphi - \nu(\sigma_r + \sigma_z) + E\epsilon_\varphi) - 2(1+\nu)\frac{\partial \tau_{rz}}{\partial z} \\ &+\frac{\partial}{\partial r}(\sigma_z - \nu(\sigma_r + \sigma_\varphi) + E\epsilon_z) = E\frac{\partial \gamma_{rz}}{\partial z}. \end{aligned} \tag{10}$$

After differentiation of Eq. (9) by z (this operation is mathematically correct because all the functions vanish when $z \to \pm\infty$) and following substitution of the expression

$$\frac{\partial^2 \sigma_z}{\partial r \partial z} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})\right), \tag{11}$$

we arrive at the following equation:

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})\right) + \frac{\partial^2 \tau_{rz}}{\partial z^2} = -\frac{1}{1+\nu}\frac{\partial^2 \sigma}{\partial r \partial z} + \frac{2G}{r}\frac{\partial}{\partial z}\left(\epsilon_r - \frac{\partial}{\partial r}(r\epsilon_\varphi)\right), \tag{12}$$

where

$$\sigma = \sigma_r + \sigma_\varphi + \sigma_z. \tag{13}$$

Note that Eq. (11) is obtained by differentiation of Eq. (3) by r.

Being derived on the basis of the first compatibility equation (4) and equilibrium equations (2) and (3), Eq. (12) can be regarded as the governing one for determination of the shear stress τ_{rz} and total stress σ. Consequently, we opt for τ_{rz} and σ to be the key functions.

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