International Journal of Pressure Vessels and Piping 88 (2011) 256–261

Contents lists available at ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

Development of an analytical reference stress equation for inner-diameter defected curved plates in tension

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ARTICLE INFO

Article history:
Received 7 April 2010
Received in revised form
13 April 2011
Accepted 13 April 2011

Keywords:
Defect assessment
Limit load
Reference stress
Curved wide plate

ABSTRACT

The tensile failure behaviour of defected structures is determined by plastic collapse and fracture. Reference stress equations can be used to predict these failure modes. Up to now, some solutions have been developed for flat plates and pipes. For curved plates, which are applied for pipe girth weld testing, however, no solutions can be found in the literature. Therefore, the authors have developed a reference stress equation that applies for curved plates with a part-through defect, located centrally along the inner diameter. The solution is global, and similar to the Goodall and Webster equation for flat plates. This article elaborates the analytical development and studies the influence of all geometrical parameters, plate curvature in particular. It is found that the solution converges to the Goodall and Webster equation for increasingly flat plates, and allows larger tensile stresses for increasingly curved plates. Hence, the proposed equation is less conservative for inner-diameter defected curved plates than Goodall and Webster's equivalent for flat plates. Nevertheless, the difference between Goodall and Webster's solution and the proposed solution is fairly restricted (less than 5% for all considered geometries). An extensive validation of the proposed equation is part of current and future work.

1. Introduction

Generally, the failure of a defected structure is governed by two different failure modes: plastic collapse and fracture. Both modes can be simultaneously investigated using a failure assessment diagram (FAD) as described in some standards and recommended practices, e.g. R6 [1], BS7910 [2], FITNET [3], API RP579 [4].

On the one hand, the calculation of proximity to plastic collapse in a FAD analysis (plotted on the horizontal axis) requires knowledge of a limit load, defined as the collapse load of the structure, assuming a perfectly plastic material. A situation of 'local collapse' can be investigated, in which case the limit load corresponds to a collapse of the ligament ahead of the defect. In contrast, 'global collapse' refers to the yielding of the entire cross section containing the defect. Completely equivalent to the concept of a limit load is the so-called reference stress. This stress is defined in such a way that, when the limit load is achieved, it reaches the metal's yield strength. The concept of reference stress assumes a perfectly plastic yielding behaviour. By definition, limit load and reference stress are connected through the following relation [5]:

$$\frac{\sigma_{ref}}{\sigma_y} = \frac{P}{P_L} \tag{1}$$

where σ_{ref} is the reference stress, σ_y the yield stress, P the applied load, and P_L the limit load. In a FAD diagram, σ_{ref}/σ_y is denoted as L_r and plotted on the horizontal axis.

On the other hand, the calculation of proximity to fracture in a FAD analysis (plotted on the vertical axis) requires knowledge of the crack driving force, expressed in terms of stress intensity factor K, crack tip opening displacement (CTOD) or J integral. K applies to linear-elastic fracture mechanics, whereas CTOD and J integral are related quantities in elastic-plastic fracture mechanics. To estimate the crack driving force, Ainsworth [5,6] started from Kumar and Shih's [7] results to obtain an expression for J integral that requires a reference stress:

$$J = \frac{K^2}{E'}\left(\frac{E\epsilon_{ref}}{\sigma_{ref}} + \frac{\sigma_{ref}^3}{2E\epsilon_{ref}\sigma_y^2}\right) \tag{2}$$

In this expression, K is the linear-elastic mode-I stress intensity factor, E' is equal to Young's modulus E for plane stress, and to $E/(1-\nu^2)$ for plane strain, where ν is Poisson's ratio. ϵ_{ref} is the reference strain, which corresponds to σ_{ref} on the stress–strain diagram of the material. A reference stress that corresponds to the

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doi:10.1016/j.ijpvp.2011.04.001

limit load through Eq. (1) can also be used in Eq. (2). This was the basis of the reference stress based FAD approach as described in R6 [1]. Numerical research by Lei [8] on flat plates with surface cracks under tension indicated that, following that procedure, global collapse solutions are better suited than local collapse solutions, which were found to be overly conservative.

Pipeline girth welds unavoidably contain defects which should be assessed to ensure the structural integrity of the pipeline. To that purpose, so-called curved wide plate tests can be performed [9,10]. A curved wide plate specimen is a large-scale sample from a pipeline, containing a (deliberately) defected weld, which is loaded in tension until failure. In an attempt to investigate curved wide plate test results, the authors concluded that, up to now, no reference stress solutions for curved plates exist. Only flat plate solutions have been found, some of which are summarized in [11], others of which can be found in [12–14]. Using those flat plate solutions as an approximation for a curved geometry, an investigation of the influence of plate curvature is impossible.

To get a better understanding of the failure behaviour of defected pipeline girth welds, the authors have developed an analytical reference stress solution for curved plates, which is elaborated in the following. The article is structured as follows. Section 2 treats the development of the equation. It starts from assumptions, similar to those of the global Goodall and Webster equation for flat plates [12]. Next, Section 3 discusses the influence of plate curvature and other geometrical parameters on the reference stress. Finally, conclusions are drawn in Section 4. Also, an Appendix is added, that briefly discusses the Goodall and Webster equation.

2. Development of the equation

2.1. Definition of geometry, material and load

The assumptions made below are similar to those made in the development of the Goodall and Webster equation [12]. This equation has been found to provide reasonable predictions of plastic collapse and J integral as a crack driving force [8,15]. For a short background on the Goodall and Webster reference stress solution, we refer to Appendix A.

Fig. 1 shows the considered plate and defect geometry. First, the curved plate is characterized by its width $2W$ (defined at

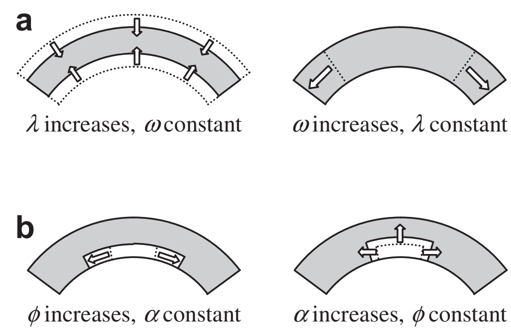

Fig. 2. Overview of influence of dimensionless parameters on geometry: (a) parameters related to plate, (b) parameters related to defect.

mid-thickness), thickness t and outer diameter D_o. Dimensionless variables which relate to these quantities are the diameter-to-thickness ratio $\lambda = D_o/t$ and the width-to-diameter ratio $\omega = 2W/D_o$. These two parameters could be considered as a measure of plate curvature, the limit case of a flat plate corresponding with $\lambda = \infty$ and $\omega = 0$. Second, the part-through defect is assumed to have a constant depth a over its length $2c$, which is measured as the distance between the two points where the defect tip meets the inner surface of the plate. The length-to-depth ratio of the defect is denoted as $\phi = 2c/a$, and defect depth is related to plate thickness using $\alpha = a/t$. To ensure conservativeness, the assumed defect should circumscribe the actual defect. The meanings of λ, ω, ϕ and α are graphically summarized in Fig. 2. Further, the parameter $\gamma = 2ca/2Wt$ is introduced as an approximate ratio of defect surface to plate cross section. It is related to λ, ω, ϕ and α as follows:

$$\gamma = \frac{\phi\alpha^2}{\lambda\omega} \tag{3}$$

Note that α and γ have a similar meaning as in the original Goodall and Webster equation.

As regards the stress distribution, the following assumptions are made. The material is considered to behave rigid – perfectly plastic. The stress in the defected section is then assumed to be $+\sigma_{ref}$ below the neutral axis, and $-\sigma_{ref}$ above. This neutral axis is oriented horizontally, and located at a distance $\bar{y}$ from the point of the

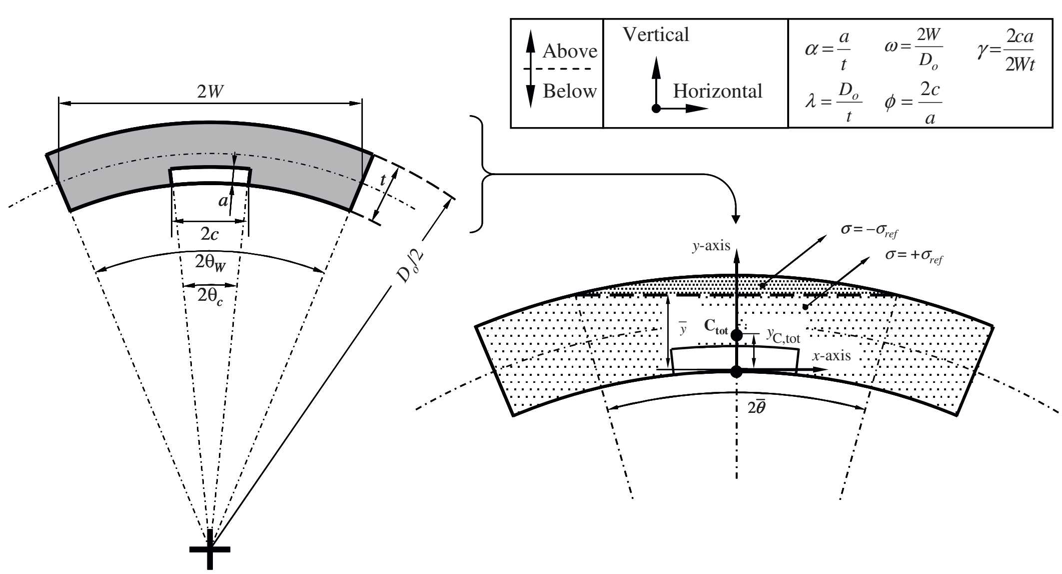

Fig. 1. Considered geometry, assumed stress distribution and definition of dimensionless parameters.