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Residual stress analysis of autofrettaged thick-walled spherical pressure vessel

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ABSTRACT

In this study, residual stress distributions in autofrettaged homogenous spherical pressure vessels subjected to different autofrettage pressures are evaluated. Results are obtained by developing an extension of variable material properties (VMP) method. The modification makes VMP method applicable for analyses of spherical vessels based on actual material behavior both in loading and unloading and considering variable Bauschinger effect. The residual stresses determined by employing finite element method are compared with VMP results and it is demonstrated that the using of simplified material models can cause significant error in estimation of hoop residual stress, especially near the inner surface of the vessel. By performing a parametric study, the optimum autofrettage pressure and corresponding autofrettage percent for creating desirable residual stress state are introduced and determined.

1. Introduction

Hydraulic autofrettage is a process in which a cylindrical or spherical pressure vessel is subjected to high internal pressure till its wall becomes partially plastic. The resulting compressive hoop residual stress produced after removing the pre-pressure improves the fatigue life and load capability of the vessel [1–3]. This specification besides a comparatively reasonable cost of the process has caused frequent and vast use of autofrettaged vessels in a variety of applications, especially in the case of high and cyclic applied internal pressure.

Spherical pressure vessels, despite difficulties in manufacturing, due to appropriate stress and strain distributions are extensively used in critical applications. Although, the study of autofrettage technique in tubes has been the subject of several research projects [4–7], autofrettage of spherical pressure vessels was only recently considered [8–13]. Adibi-asl and Livieri [8] proposed an analytical method for residual stress analyses of autofrettaged spherical vessels. Their investigation is limited to materials which follow a constrained form of modified Ramberg-Osgood, Isotropic or kinematic hardening models during loading and unloading. Kargarnovin et al. [10] applied an elastic-linear work hardening model and ignored the Bauschinger effect to evaluate optimum prestressing pressure in a spherical vessel. While the response of the material models applied in these investigations are not compatible with the actual behavior of several of the most used materials in pressure vessels [14,15], their results can not be reliably employed. Parker and Huang [12] put forward analytical solution of autofrettaged spherical vessels incorporating more sensible material model and verified their established method by comparing the results of developed variable material property method at particular autofrettage percents. Perl and Perry [13] extended the existing knowledge to provide more realistic solutions for the residual stress fields in thick-walled autofrettaged spherical pressure vessels. Applying the minimum weight and the maximum life criterion, they also proposed an optimum design of autofrettage process.

It is thanks to previous research efforts that a proper approximation of residual stress distribution in spherical pressure vessels is achievable now, but still need to be more investigated.

To analyze the residual stress distribution induced by autofrettage process of a thick-walled spherical pressure vessel, we employ an extension of the variable material property (VMP) method introduced by Jahed and Dubey [16] and compared the obtained results with the results of finite element analyses. In the VMP method, the linear elastic solution of a boundary value problem is used as a basis to generate its inelastic solution. The material parameters are considered as field variables and their distribution is obtained as a part of solution in an iterative manner. This method was generally employed to homogeneous and inhomogeneous cylindrical pressure vessel and rotating disks [17–21]. The proposed extension makes the VMP method capable for

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analyzing the residual stresses in autofrettaged spherical pressure vessels. The main advantage of this method is its capability in the implementation of actual behavior of material obtained from loading and unloading tests in the analyses.

In addition, the optimum pressure for autofrettage of the spherical pressure vessels made of A723 and HB7 steels is discussed here. In this study, an autofrettage percent has been defined as the optimum value by which the significant compressive hoop residual stress at inner surface of the vessel is achievable, while simultaneously the tensional hoop residual stress at outer surface is relatively low.

2. Theoretical model

The components of total strain of an infinitesimally small element located at a distance r from the center of a thick-walled sphere can be represented as summation of elastic, ε_{ij}^{e} and plastic, ε_{ij}^{p}, strain components. The elastic part is given as,

$$\varepsilon_{ij}^{e} = \left(\frac{1+\nu}{E}\right)\sigma_{ij} - \left(\frac{\nu}{E}\right)\sigma_{kk}\delta_{ij} \tag{1}$$

where δ_{ij}, E, ν are the Kronecker delta, elastic modulus and Poisson's ratio respectively. The plastic part of strain is given by Hencky's total deformation equation, $\varepsilon_{ij}^{p} = \phi(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij})$ where ϕ is a scalar function given by $\phi = \frac{3}{2}\frac{\varepsilon_{eq}^{p}}{\sigma_{eq}}$, where ε_{eq}^{p} and σ_{eq} are the equivalent plastic strain and equivalent stress, respectively. Consequently the total strain in the element can be written as,

$$\varepsilon_{ij} = \left(\frac{1+\nu}{E} + \phi\right)\sigma_{ij} - \left(\frac{\nu}{E} + \frac{1}{3}\phi\right)\sigma_{kk}\delta_{ij} \tag{2}$$

In order to analyze a thick-walled spherical vessel, we divide the vessel into a number of concentric thin spheres (layers) through the thickness, where the inner radius of the interior layer and the outer radius of exterior layer are equal to interior and exterior radii of the vessel.

According to VMP approach, the total strain components of each elasto-plastic layer can be calculated by the following equation:

$$\varepsilon_{ij} = \frac{1+\nu_{eq}(r)}{E_{eq}(r)}\sigma_{ij} - \frac{\nu_{eq}(r)}{E_{eq}(r)}\sigma_{kk}\delta_{ij} \tag{3}$$

where $\nu_{eq}(r)$ and $E_{eq}(r)$ are equivalent Poisson's ratio and equivalent elastic modulus of a layer with distance of r from the center of sphere. Comparing equations (2) and (3), the following relationships are obtained:

$$E_{eq}(r) = \frac{3E}{3+2E\phi} \tag{4}$$

$$\nu_{eq}(r) = \frac{E_{eq}(r)(2\nu-1)+E}{2E} \tag{5}$$

Based on the VMP method, the elasto-plastic solution is achievable by implementation of appropriate variable material constants and performing the equivalent elastic analysis.

The elastic solution of a spherical layer with inner and outer radii of r_i and r_{i+1} and material constants of ν_{eq} and E_{eq} is:

$$u = \frac{\left(p_i r_i^3 - p_{i+1} r_{i+1}^3\right)\left(1+\nu_{eq}\right)\left(1-2\nu_{eq}\right)}{E_{eq}\left(1-\nu_{eq}\right)\left(r_{i+1}^3 - r_i^3\right)} r + \frac{r_i^3 r_{i+1}^3 \left(p_i - p_{i+1}\right)\left(1+\nu_{eq}\right)}{2E_{eq}\left(r_{i+1}^3 - r_i^3\right) r^2} \tag{6}$$

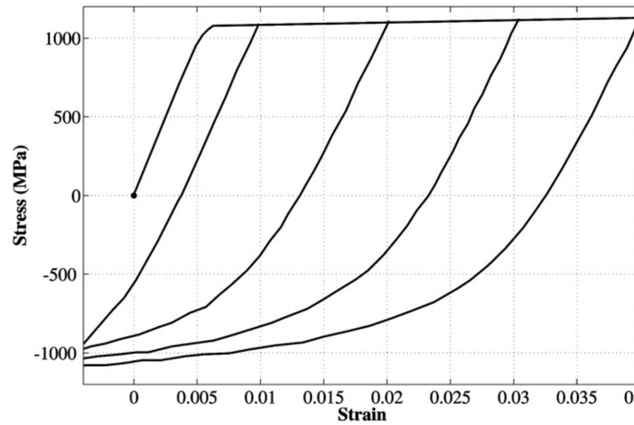

Fig. 1. Engineering stress-strain curve of A723 steel, after Toriano et al. [23].

$$\sigma_{rad} = \frac{p_i r_i^3 - p_{i+1} r_{i+1}^3}{r_{i+1}^3 - r_i^3} - \frac{r_i^3 r_{i+1}^3 \left(p_i - p_{i+1}\right)}{r^3\left(r_{i+1}^3 - r_i^3\right)}$$

$$\sigma_\theta = \sigma_\phi = \frac{p_i r_i^3 - p_{i+1} r_{i+1}^3}{r_{i+1}^3 - r_i^3} + \frac{r_i^3 r_{i+1}^3 \left(p_i - p_{i+1}\right)}{2r^3\left(r_{i+1}^3 - r_i^3\right)} \tag{7}$$

where p_i and p_{i+1} are internal and external pressures, u is radial displacement and σ_{rad}, σ_θ and σ_ϕ are stress components.

Based on equation (6), the inside and outside displacements of each layer located at r, u_i and u_{i+1}, are related to the inside and outside pressures by:

$$\begin{bmatrix} c_{11,r} & c_{12,r} \\ c_{21,r} & c_{22,r} \end{bmatrix}^{-1} \begin{bmatrix} u_i \\ u_{i+1} \end{bmatrix} = \begin{bmatrix} p_i \\ p_{i+1} \end{bmatrix} \tag{8}$$

where:

$$c_{11,r} = \frac{r_i\left[\left(1-2\nu_{eq}(r)\right)r_i^3 + r_{i+1}^3\right]}{E_{eq}(r)\left(r_{i+1}^3 - r_i^3\right)}$$

$$c_{21,r} = \frac{2r_{i+1} r_i^3 \left(1-\nu_{eq}(r)\right)}{E_{eq}(r)\left(r_{i+1}^3 - r_i^3\right)}$$

$$c_{12,r} = \frac{2r_i r_{i+1}^3 \left(\nu_{eq}(r)-1\right)}{E_{eq}(r)\left(r_{i+1}^3 - r_i^3\right)}$$

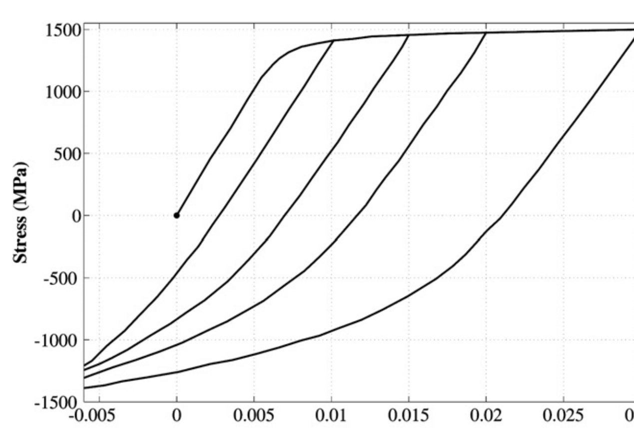

Fig. 2. Engineering stress-strain curve of HB7 steel, after Troiano et al. [22].