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A multi-point univariate decomposition method for structural reliability analysis

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ABSTRACT

A new multi-point univariate decomposition method is presented for structural reliability analysis involving multiple most probable points (MPPs). The method involves a novel function decomposition at all MPPs that facilitates local univariate approximations of a performance function in the rotated Gaussian space, Lagrange interpolation for univariate component functions and return mapping to the standard Gaussian space, and Monte Carlo simulation. In addition to the effort in identifying all MPPs, the computational effort in the multi-point univariate method can be viewed as performing deterministic response analysis at user-selected input defined by sample points. Compared with the existing multi-point FORM/SORM, the multi-point univariate method developed provides a higher-order approximation of the boundary of the failure domain. Both the point-fitted SORM and the univariate method entail linearly varying cost with respect to the number of variables. However, the univariate method with less than nine sample points requires fewer calculations of the performance function than the point-fitted SORM. Numerical results indicate that the proposed method consistently generates an accurate and computationally efficient estimate of the probability of failure.

1. Introduction

Structural reliability analysis frequently involves calculation of a component probability of failure

$$P_F \equiv P[g(\mathbf{X})<0] = \int_{g(\mathbf{x})<0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}, \quad (1)$$

where $\mathbf{X} = \{X_1,\ldots,X_N\}^T \in \mathbb{R}^N$ is a real N-dimensional random vector defined on a probability space $(\Omega, \mathcal{F}, P)$ comprising the sample space Ω, the σ-field $\mathcal{F}$, and the probability measure P; $g:\mathbb{R}^N \to \mathbb{R}$ is a performance function, such that $\Omega_F \equiv \{\mathbf{x} : g(\mathbf{x})<0\}$ represents the failure domain; and $f_{\mathbf{X}}:\mathbb{R}^N \to \mathbb{R}$ is the joint probability density function of $\boldsymbol{X}$, which typically represents loads, material properties, and geometry. The most common approach to compute the failure probability in Equation (1) involves the first- and second-order reliability methods (FORM/SORM) [1–3], which are respectively based on linear (FORM) and quadratic (SORM) approximations of the limit-state surface at a most probable point (MPP) in the standard Gaussian space. When the distance β between the origin and MPP (a point on the limit-state surface that is closest to the origin), known as the Hasofer-Lind reliability index, approaches infinity, FORM/SORM provide strictly asymptotic solutions. For non-asymptotic (finite β) applications involving a highly nonlinear performance function, its linear or quadratic approximation may not be adequate and, therefore, resultant FORM/SORM predictions should be interpreted with caution [4,5]. In latter cases, an importance sampling method developed by Hohenbichler and Rackwitz [6] can make FORM/SORM result arbitrarily exact, but it may become expensive if a large number of costly numerical analysis, such as large-scale finite element analysis embedded in the performance function, are involved. In addition, if multiple MPPs exist in either asymptotic or non-asymptotic applications, or if there are contributions from other regions around local minima besides the region around a single MPP, classical FORM/SORM may yield erroneous estimates of the failure probability [7–10]. Therefore, methods that can account for both sources of errors due to high nonlinearity and multiple MPPs are required for structural reliability analysis.

For reliability problems entailing multiple MPPs, the failure probability can be estimated by the multi-point FORM/SORM, which leads to a probability of the union of approximate events [3]. Der Kiureghian and Dakessian [7] proposed a so-called "barrier" method to successively find multiple MPPs. Subsequently, FORM/SORM approximations at each MPP followed by a series system reliability analysis were employed to estimate the failure probability. While the multi-point FORM/SORM account for all MPPs, the resultant effects are limited to first- or second-order approximations of the

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performance function. More recently, Au et al. [8] presented asymptotic approximations and importance sampling methods for solving reliability problems with multiple MPPs. Mahadevan and Shi [9] proposed a multiple linearization method in which the limit-state surface is approximated using multiple linear hyperplanes. However, for a general reliability analysis involving a large number of random variables, it is difficult to determine the number of linearization points and locate them systematically. Gupta and Manohar [10] proposed a global response surface method, which constructs a response surface of the limit state by using the global information, rather than the local information around a single MPP. This method requires defining a new set of coordinates and a number of shifting origins in advance. If the performance function is implicit and/or the number of random variables is very large, it is difficult to apply this strategy. Recently, the authors have developed a new class of reliability methods, called the mean- [5] and MPP-based [11,12] dimensional decomposition methods, which are based on a finite hierarchical expansion of the performance function in terms of input variables with increasing dimension. Although these decomposition methods provide higher-order approximations of a performance function, they cannot account for multiple MPPs [11,12]. Hence, developing a multi-point decomposition method in the spirit of the multi-point FORM/SORM that accounts for high nonlinearity and multiple MPPs is the principal motivation of this work.

This paper presents a new multi-point univariate decomposition method for predicting component reliability of mechanical systems subject to random loads, material properties, and geometry. Section 2 gives a brief exposition of novel function decomposition at an MPP that facilitates a lower-dimensional approximation of a general multivariate function. Section 3 describes the proposed univariate method that involves local univariate approximations of the performance function with multiple MPPs, Lagrange interpolation of univariate component functions, return mapping, and Monte Carlo simulation. The section also explains the computational effort and flowchart of the proposed method. Three numerical examples involving elementary mathematical functions and a structural dynamics problem illustrate the method developed in Section 4. Finally, Section 5 provides conclusions from this work.

2. Performance function decomposition at the *m*th MPP

Consider a continuous, differentiable, real-valued performance function $g(\mathbf{x})$ that depends on $\mathbf{x} = \{x_1, \ldots, x_N\}^T \in \mathbb{R}^N$. The transformed limit state $h(\mathbf{u}) = 0$ is the map of $g(\mathbf{x}) = 0$ in the standard Gaussian space ($\mathbf{u}$ space), as shown in Fig. 1 for $N = 2$. Let the performance function contain M number of MPPs $\mathbf{u}_1^*, \ldots, \mathbf{u}_M^*$ with corresponding distances $\beta_1, \ldots, \beta_M$ (Fig. 1).

For the mth MPP, define an associated local coordinate system $\mathbf{v}_m = \{v_{m,1}, \ldots, v_{m,N}\}$, where $v_{m,N}$ is the coordinate in the direction of the MPP, as depicted in Fig. 1. In the $\mathbf{v}_m$ space, denote the mth MPP by $\mathbf{v}_m^* = \{0, \ldots, 0, \beta_m\}$ and the limit state surface by $y_m(\mathbf{v}_m) = 0$, which is also a map of the original limit state surface $g(\mathbf{x}) = 0$. The decomposition of a general multivariate function $y_m(\mathbf{v}_m)$, described by [11–17]

$$y_m(\mathbf{v}_m) = \underbrace{\underbrace{y_{m,0} + \sum_{i=1}^{N} y_{m,i}(v_{m,i})}_{=\widehat{y}_{m,1}(\mathbf{v}_m)} + \sum_{\substack{i_1,i_2=1\\ i_1<i_2}}^{N} y_{m,i_1i_2}(v_{m,i_1}, v_{m,i_2})}_{=\widehat{y}_{m,2}(\mathbf{v}_m)} + \ldots + y_{m,12\ldots N}(v_{m,1}, \ldots, v_{m,N}), \quad (2)$$

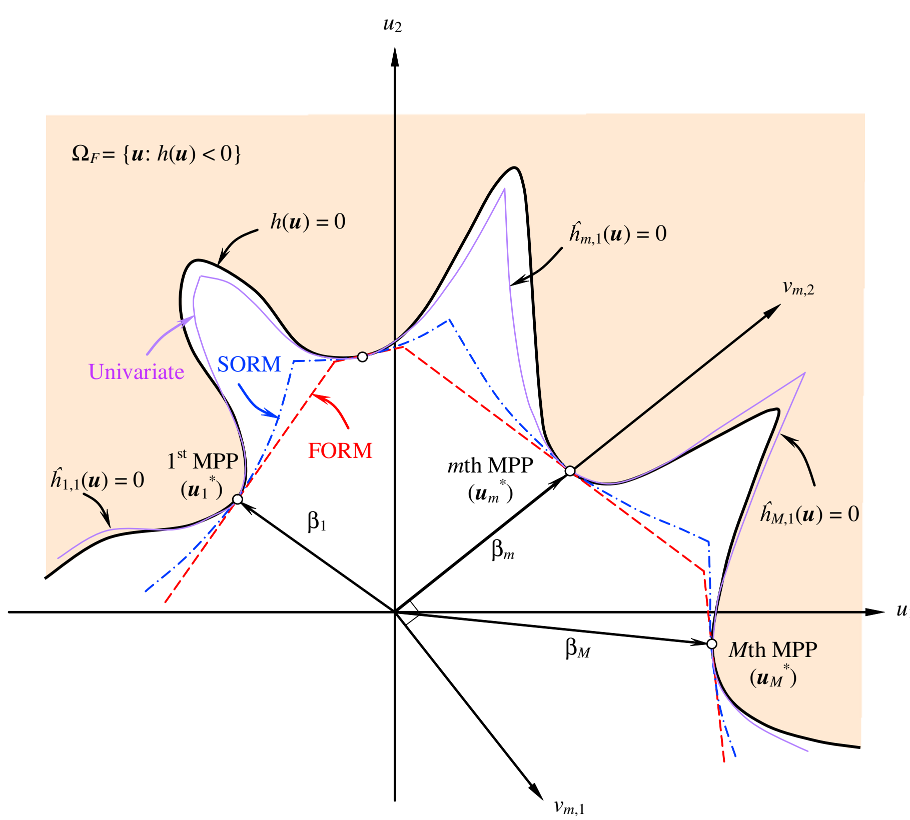

Fig. 1. A performance function with multiple most probable points.