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# Elastic–plastic behaviours of pressurised tubes under cyclic thermal stresses with temperature gradients

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ABSTRACT

Elastic–plastic behaviours of pressurised tubes under cyclic thermal stresses are investigated, incorporating temperature (and resulting the yield strength) gradient through the tube thickness. Based on analytical investigations, explicit equations for various stress regimes representing elastic–plastic behaviours are obtained. In the limiting case of no temperature (and yield strength) gradient through the thickness, proposed equations recover those for the well-known Bree diagram. The proposed results are then validated against the results from full-cyclic elastic–plastic finite element analysis.



## 1. Introduction

Pressurised tubes in power and chemical plants are often subjected to repeated thermal loads. When the loading of such components is severe, then cyclic plastic straining or gross distortion (ratchetting) could occur. Thus knowledge of the load conditions to avoid ratchetting is required in the design of such components. In obtaining analytical solutions for elastic–plastic behaviours of pressurised tubes under cyclic thermal stresses, Bree [1,2] used a uni-axial model to simplify the problem, of which the result is now well-known as the Bree diagram (see ASME [3]). In his analysis, however, through-thickness variations of radial and hoop stresses (due to the application of internal pressure) and of axial loading (which may be caused by closed ends or axial restraint, for example) were not considered, which were systematically investigated via detailed finite element analyses [4–8]. However, the above analyses [1–8] assume no temperature variation across the tube thickness during operation/shutdown. Thus no temperature gradient exists across the tube thickness in the operating condition. Furthermore, the previous analyses are based on the assumption that the yield strength is constant over the entire thermal cycle. Noting that yield strengths of materials are typically dependent on temperature, the effect of the temperature-dependent yield strength was also incorporated into elastic–plastic behaviours of pressurised tubes under cyclic thermal stresses [8,9].

In certain applications, however, the tube inner wall temperature is different from that at the outer wall during operation; hence a temperature gradient exists across the tube thickness. Typical examples include, for instance, heat exchangers [10] and co-axial tubes [11]. In such cases, yield strengths through the thickness can be different due to temperature gradient, which makes elastic–plastic behaviours of pressurised tubes different from existing ones.

This paper investigates elastic–plastic behaviours of a pressurised tube under cyclic thermal stresses with temperature gradient through the tube thickness. An emphasis is given on the temperature gradient through the thickness and the resulting variations in yield strengths. Section 2 sets up a tube geometric model and loading, considered in the present work, together with assumptions. Explicit equations for various stress regimes representing elastic–plastic behaviours are given in Section 3. Section 4 validates the proposed results using full-cyclic elastic–perfectly plastic FE analysis. The present work is concluded in Section 5.

## 2. Tube geometric model, loading and assumptions

This paper considers a pressurised tube subject to cyclic temperature variations, as schematically depicted in Fig. 1. The

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**Nomenclature**

| | |
|---|---|
| $E$ | Young's modulus |
| $T$, $\Delta T$ | temperature and its increment, respectively |
| $P$ | internal pressure |
| $X$ | normalized primary stress, see Eq. (4) |
| $Y$ | normalized thermal stress, see Eq. (4) |
| $a$, $b$ | location of the elastic–plastic interface |
| $r$ | mean tube radius |
| $w$ | half-thickness of the tube |
| $\alpha$ | thermal expansion |
| $\psi$ | non-dimensional variable, $=\sigma_{YL}/\sigma_{YH}$ |
| $\sigma_Y$ | yield strength (general) |
| $\sigma_{YH}$ | yield strength at the higher temperature, $T = T_o + \Delta T$ |
| $\sigma_{YL}$ | yield strength at the lower temperature, $T = T_o$ |
| $\sigma_p$ | primary stress due to internal pressure |
| $\sigma_t$ | thermal stress due to temperature gradient |

mean radius and thickness of the tube are denoted by $r$ and $2w$, respectively, and the tube is subject to constant internal pressure $P$. The tube is assumed to be subject to cyclic thermal stresses as follows (see also Fig. 1). Initially the tube is subject to a constant temperature $T_o$ both at the inner and at the outer wall. The start-up and operating conditions are that the temperature at the inner wall increases to $T_o + \Delta T$ ($\Delta T > 0$), and remains constant. The temperature at the outer wall, on the other hand, remains constant as $T_o$. The shutdown condition is the same as the initial condition of constant temperature $T_o$ at both inner and outer walls. This thermal loading cycle is repeated. The objective of this problem is to quantify the effect of the cyclic temperature gradient through the tube thickness on elastic–plastic behaviours of pressurised tubes. However, it should be pointed out that the problem is highly idealised in the sense that temperature at the outer wall is assumed to be constant at start-up and shutdown. Consideration of temperature changes at the outer wall could add much more complexities to the problem, and thus is neglected in the present work.

As the yield strength of a material should depend on temperature, it should vary across the thickness under the steady-state temperature condition. Let us denote the yield strength at temperature $T_o$ as $\sigma_{YL}$ and that at $(T_o + \Delta T)$ as $\sigma_{YH}$. Introducing the non-dimensional variable $\psi$, defined by the ratio of the yield strengths:

$$\psi = \frac{\sigma_{YL}}{\sigma_{YH}} \tag{1}$$

The value of $\psi = 1$ corresponds to the case of the constant yield strength across the thickness, and thus to the case of the temperature-independent yield strength. As the yield strength typically decreases with increasing temperature, the value of $\psi$ should be greater than unity, $\psi \geq 1.0$.

For simplicity, following assumptions are made in this work:

(i) Material is assumed to be elastic–perfectly plastic.
(ii) The length of the tube is sufficiently long so that the end effect is negligible.
(iii) The tube is assumed to be thin and open-ended, and thus only the non-zero strength component is the hoop strength (both axial and radial stresses are zero).
(iv) Temperature distribution along the thickness is assumed to be linear.
(v) Yield strength decreases linearly with increasing temperature.
(vi) The small displacement (strain) assumption is made and thus the non-linear geometry effect is neglected.

Note that the last assumption of linear dependence of the yield strength on temperature is imposed to analytically develop diagrams for elastic–plastic behaviours. However, it will be shown that such an assumption is not central, and the proposed method can be applied to any functional dependence of the yield strength on temperature, as will be shown in Section 4.

## 3. Proposed stress regimes

In this section, boundaries of stress regimes for elastic–plastic behaviours of pressurised tubes under cyclic temperature gradients through the thickness will be derived in dimensionless forms. For compact notation, non-dimensional variables are introduced. The tube is subject to the primary stress, $\sigma_p$ (due to internal pressure), and the secondary (thermal) stress, $\sigma_t$ (due to temperature gradient), given by

$$\sigma_p = \frac{P \cdot r}{2w}$$
$$\sigma_t = \frac{E\alpha\Delta T}{2}\left(\frac{x}{w}\right) \left(-1 \leq \frac{x}{w} \leq 1\right) \tag{2}$$

The total stress in the tube can be found simply by linear superposition:

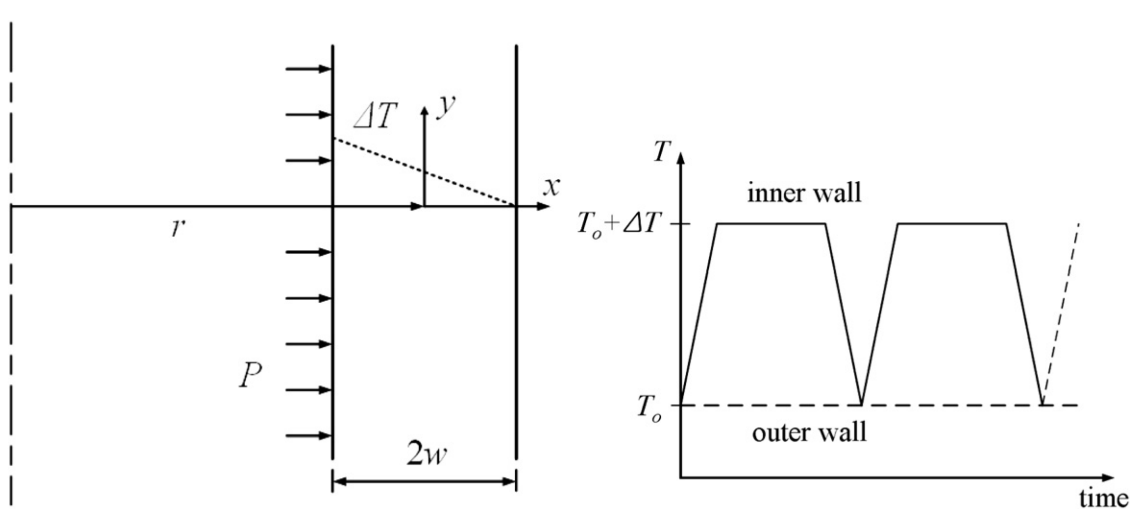


**Fig. 1.** A schematic diagram of a pressurised tube with linear temperature gradient through the thickness.