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Response analysis of nonlinear vibro-impact system coupled with viscoelastic force under colored noise excitations

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ABSTRACT

This paper is mainly dealing with the stochastic responses of nonlinear vibro-impact (VI) system coupled with viscoelastic force excited by colored noise. By the aid of approximate conversion for the viscoelastic force, the original stochastic VI system is transformed into an equivalent stochastic system without viscoelastic term. Then, the equations of the converted system are simplified by non-smooth transformation, and the stochastic averaging method is employed to solve the above simplified system. A Van der Pol VI oscillator coupled with viscoelastic force is worked out in detail to illustrate the application of the mentioned method, and therewith the analytical solutions fit the numerical simulation results based on the original system. Therefore, the present analytical means of investigating this system is proved to be feasible. Additionally, the exploration of stochastic P-bifurcation by two different ways is also demonstrated in this paper through varying the value of the certain system parameters. Besides, it shows a noteworthy fact that assigning zero or a positive value to the magnitude of viscoelastic force can also lead to the bimodal shape of different degrees in the process of stochastic bifurcations.

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1. Introduction

Viscoelasticity of engineering materials is a kind of property which is preferred. Impact is also inevitable in nature, technology and society. Therefore, considering the effects of viscoelastic behavior in VI system play a significant role in understanding the physical mechanism of control systems in the real world, and is of interest in the field of nonlinear science.

Impact under dynamic loads is a dynamic phenomenon produced by the repeatedly contact among the mechanical components with clearance or kinematical constraint, which means that a lumped mass impacts a rigid barrier with finite velocity. As a type of non-smooth system, VI system received much attention in recent years. We can see the impact factor contributes to some fascinating phenomena for dynamic system, such as corner bifurcation [1], grazing bifurcation [2–4], sliding bifurcation [5,6], chatter and sticking motions [7,8].

In some of the existing literature, the research result of the vibro-impact systems is quite abundant. Nordmark [9] devoted to studying the dynamical characteristics of vibro-impact oscillator, then obtained the corresponding Jacobin array via the Poincaré map, and also observed the stationary feature of the non-smooth system. Aidanpää [10] investigated successively the stability and bifurcations of the stationary state for a one-degree-of-freedom vibro-impact oscillator. Besides, many attentions have been draw to stochastic response of vibro-impact system, and some effective means appear gradually. Quasi-static method has been adopted by Stratonovich [11]. The non-smooth transformation was introduced by Zhuravlev [12] for vibro-impact system with rigid barriers. Some scholars [13–17] extended the stochastic averaging to deal with the vibro-impact system. Additionally, a modified version of quasi-conservative averaging was proposed by Roberts [18] to solve the stochastic systems involving a non-white random excitations. By applying the Poincaré map, Luo [19] and Xie [20] considered bifurcations as well as chaos of a two-degree-of-freedom linear vibro-impact systems. Feng [21,22] investigated the mean response of impact systems by introducing the mean Poincaré map.

Viscoelasticity of materials in engineering practice shows that energy storage as well as mechanical energy dissipation, which is a superior characteristic and has been favored by many scholars. Fortunately, scholars have been dedicated to exploring a constitutive model describing the viscoelastic property, and then some models [23–25] have been put forward and developed in the last couple of years, which is roughly divided into differential model and integral model. Further, a simple linear viscoelastic model based on the generalized Maxwell model [26–28] has been widely used. In the investigation of practical phenomena,

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stochastic perturbation is inevitable [29], and the study of dynamic behaviors of viscoelastic system has experienced a developmental process from deterministic case [30–33] to stochastic one.

Clearly, the stochastic viscoelastic systems have gradually caused attention among researchers. For instance, Spanos [34] has demonstrated the response of a class of oscillators with non-linear damping to stochastic excitation. Potapov [35] analyzed the stability of stochastic viscoelastic systems by stochastic averaging. Zhu and Cai [36] analyzed random vibration of viscoelastic system via generalized Maxwell model and quasi-conservative averaging method, and pointed out that the magnitude of viscoelastic force can be positive and negative, then gave the conclusion of the influence of the magnitude taking changing negative values on the response of the system. Zhao and Xu et al. [37,38] considered stochastic responses of viscoelastic system under Gaussian white noise excitation and discussed stochastic bifurcation induced by viscoelastic parameters, in which the magnitude has been only gotten the negative value. So the situation of the magnitude taking a non-negative value deserves some consideration.

Besides, Potapov [39] applied Lyapunov's direct method to study the almost-sure stability of a viscoelastic column excited by a random wideband stationary process. Potapov [40] also analyzed stability of elastic and viscoelastic systems under non-Gaussian excitation by the numerical method. Xie [41] investigated the moment Lyapunov stability of a two-dimensional viscoelastic system subject to bounded noise excitation. Floris [42] explored the stochastic stability of a hinged-hinged viscoelastic column. The transient response of linear viscoelastic systems with model uncertainties and stochastic excitation by a time-domain formulation has been suggested by Soize and Poloskov [43]. Stochastic stability of the harmonically and randomly excited Duffing oscillator with damping modeled by a fractional derivative was surveyed by Chen [44]. Ling [45] discussed the stability of a viscoelastic system under wideband noise by means of the largest Lyapunov exponent. It can also be found that the viscoelastic property of materials has great significance for the research and practice, and deserves more attention.

The occurrence of impacts in practical engineering often leads to some negative effects, and while viscoelastic force can suppress the vibration of the impact structures with the aid of its characteristic. Inspired by this, we consider the dynamical behavior of the VI system with viscoelastic force. In the stochastic case, there is still less clear research on the nonlinear VI system together with viscoelastic behavior excited by colored noise at present. As far as the authors know, the bimodal shape in the process of stochastic bifurcations caused by the non-negative magnitude have not yet been found in the current existing literatures which study the dynamical behaviors of stochastic system with the viscoelastic force and impacts. In this paper, we are committed to dealing with the system mentioned above, and the stationary probability density functions (PDFs) of which are discussed in detail. The rest of this paper is arranged as follows. System setup and description, non-smooth treatment and stochastic averaging are reported in Section 2. Then, a Van der Pol VI system is given to illustrate the application of the method in Section 3. Furthermore, the means to study this system turns out to be effective through comparing the analytical solutions with the numerical simulation results, and stochastic bifurcations are obtained the exploration by two angles. Conclusions are performed in the last section.

2. System setup and the method

The model adopted in this paper is a single degree of freedom nonlinear VI system coupled with viscoelastic force subjected to Gaussian colored noise excitations, which is dominated by the following equations

$$\ddot{x} + \varepsilon\mu h(x,\dot{x})\dot{x} + \omega_0^2 x + \gamma Z = \varepsilon^{1/2}\kappa_1\eta(t) + \varepsilon^{1/2}\kappa_2 x^2\xi(t),\ x > 0, \tag{1a}$$

$$\dot{x}_+ = -e_0\dot{x}_-,\ x = 0. \tag{1b}$$

Here $h(x,\dot{x})$ represents the nonlinear function of x and $\dot{x}$, ε is a small scale parameter, μ, γ and ω_0 are positive constants. κ_1, κ_2 indicate the coefficients of noise excitations, the symbols $\eta(t)$ and $\xi(t)$ denote the Gaussian colored noise with zero mean and correlations

$$\langle\eta(t)\eta(s)\rangle = \frac{D_1}{\tau_1}\exp\left(-\frac{|t-s|}{\tau_1}\right), \tag{2a}$$

$$\langle\xi(t)\xi(s)\rangle = \frac{D_2}{\tau_2}\exp\left(-\frac{|t-s|}{\tau_2}\right), \tag{2b}$$

$$\langle\eta(t)\xi(s)\rangle = 0, \tag{2c}$$

where D_1, D_2 and τ_1, τ_2 denote the intensities and correlation times of the colored noises $\eta(t)$ and $\xi(t)$, respectively. In Eq. (1b), $0 < e_0 \le 1$ is the coefficient of restitution factor, whose value reflects the degree of energy loss of the system when impact occurs, and signs "$\dot{x}_-$" and "$\dot{x}_+$" refer to value of response velocity just before and after the impact. Thus, Eq. (1b) provides the reversal of velocity $\dot{x}$ at the time instant of impact t_o. Under the condition of $e_0 = 1$, this special case is modeled as elastic impacts with getting the exceedingly small energy loss.

Besides, Z represents viscoelastic force in Eq. (1a), whose description is exhibited as the following form:

$$Z[x(t)] = \int_0^t I_0(t-\tau_s)x(\tau_s)d\tau_s, \tag{3}$$

where $I_0(t)$ is known as the relaxation function, and can be commonly characterized by the generalized Maxwell model

$$I_0(t) = \sum_i \beta_i e^{(-t/\lambda_i)},\ \lambda_i \ge 0, \tag{4}$$

here $I_0(t)$ is composed of a number of relaxation domains, that is, i stands for the first i relaxation domain ($i = 1, 2, 3, \ldots$). Each λ_i is named as the relaxation time of each small component, and β_i denotes general elastic modulus which may be either positive or negative. Both parameters can be determined by the specific problems.

Based on Liu and Zhu's theory [46], assuming that the coefficients of all excited terms are proportional to small parameters, substitution of Eq. (4) into Eq. (3) will yield an approximate expression for Z:

$$\begin{aligned}Z[x(t)] &= \int_0^t I_0(t-\tau_s)X(\tau_s)d\tau_s = \int_0^t I_0(\varsigma)X(t-\varsigma)d\varsigma \\ &= \int_0^t \sum_i \beta_i e^{(-\varsigma/\lambda_i)}\left(-\frac{1}{\bar{\omega}}\dot{X}\ \sin\ \bar{\omega}\varsigma + X\ \cos\ \bar{\omega}\varsigma\right)d\varsigma \\ &= \sum_i\left\{-\frac{\lambda_i^2\beta_i}{1+\bar{\omega}^2\lambda_i^2}\dot{X} + \frac{\lambda_i\beta_i}{1+\bar{\omega}^2\lambda_i^2}X\right. \\ &\quad + \frac{\lambda_i e^{(-t/\lambda_i)}}{1+\bar{\omega}^2\lambda_i^2}\left[\frac{\beta_i}{\bar{\omega}}(\sin\ \bar{\omega}t + \lambda_i\bar{\omega}\ \cos\ \bar{\omega}t)\dot{X}\right. \\ &\left.\left.+\beta_i(\lambda_i\bar{\omega}\ \sin\ \bar{\omega}t - \cos\ \bar{\omega}t)X\right]\right\}.\end{aligned} \tag{5}$$

where $\bar{\omega}$ is the average frequency according to Zhu and Cai [36], which is determined by $\bar{\omega} = \frac{2\pi}{T}$.

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