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## International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm



# Interval analysis of nonlinear frames with uncertain connection properties



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#### ARTICLE INFO

Article history: Received 30 November 2015 Received in revised form 16 April 2016 Accepted 21 July 2016 Available online 25 July 2016

Keywords: Elastoplasticity Hardening Interval analysis Mathematical programming Semi-rigid connections Steel structures

#### ABSTRACT

Semi-rigid connections can often be a more economical solution for a framing system than one with either fully fixed connections or fully pinned ones. In view of the fact that the properties of such ductile and partial-strength connections are not known accurately, this paper presents a method for the obtention of both upper and lower bound responses of semi-rigid frames for possible variations in their moment-rotation properties. The latter are thus assumed to be known within some key upper and lower bound values, namely a constitutive law that is still deterministic but is described in terms of a so-called "interval" model. A mathematical programming approach is used to formulate and solve the problem. In particular, for each load level, a pair of nonstandard optimization problems known as interval mathematical programs with equilibrium constraints (or interval MPECs) are solved to provide the required bounds. A number of examples are provided to highlight the important effects of considering uncertainties in semi-rigid connection properties.

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#### 1. Introduction

Semi-rigid beam-to-column connections have been widely employed in the design and construction of various steel structures ranging from low-rise to high-rise buildings. Such joints partially transfer flexural and shear forces between beams and columns, and are typically represented by a partial-strength, but still ductile, moment versus relative rotation relationship (see e.g. [1–3]). The safe design and analysis of these structures require an accurate description of the intrinsic (moment versus rotation) connection behavior [4,5].

The numerical simulation of semi-rigid connections and frames, especially under complex (e.g. fire, high-cycle, non-proportional, and so on) loading environments where the costs associated with actual experimental setups are fairly expensive, has gained popularity (e.g. [6–8]). Obviously, reliable results can only be obtained if the joint models used are accurate, since different models can lead to significant variation in frame responses.

Extensive research (e.g. [2,5,9–11]) has been devoted to the development of mathematical models, largely based on experimental data, that predict the behavior of various types of semi-

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rigid connections. These models invariably form the basis for the safety analysis of semi-rigid structures, especially when joint data are not available [12,11]. Even with experimental semi-rigid connection data, there is often a scatter in the results for the same connection type.

An example of the variety of models proposed for the same connection is shown in Fig. 1 for the common bolted single webangle joint [13]. The curves represent the modified exponential model (dash-dot line) [2], the polynomial model (dotted line) [3], and the power model (dashed line) [1].

As is often stated, a structure is only as strong as its weakest link—the semi-rigid connections in this instance. It is therefore crucial for engineers to assess the structural response under possible variations in semi-rigid connection properties. Moreover, the scatter in experimental data and the variations in numerical connection models for the same joint make it difficult for even unexperienced designers to decide which model to use. At the very least there should be an assessment of the effect of connection model on the structure response. This forms the basis of the present paper.

The influences of uncertainties in semi-rigid connection properties on the failure behavior of practical steel structures have been studied in the context of reliability analysis (see e.g. [14,15]). Most of the techniques used adopt either exhaustive Monte Carlo simulations or stochastic finite element analyses to determine the probability of failures of the considered structural system under

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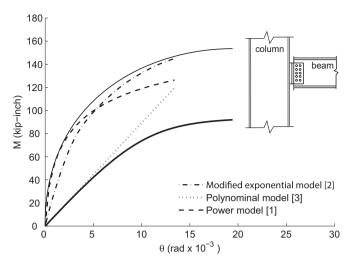


Fig. 1. Semi-rigid connection approximate models.

variations of initial moment-rotation stiffnesses of the partially restrained connections. The accuracy of the results often relies on the precision of the adopted statistical uncertainty data, describing the variability of the intrinsic connection behaviors. When the sources of uncertainties can be approximated by the two extreme upper and lower bound values, the probabilistic description reduces to a simpler form, known as a "convex" model [16]. An interval finite element method [17] was proposed to obtain the corresponding critical interval responses for static (linear) mechanics problems, including ones involving interval variations of initial connection stiffnesses.

The present study develops an analysis method suitable for obtaining the extreme responses of a semi-rigid frame, assuming that the upper and lower bound moment-rotation curves are known, for instance, as estimated by the two solid (bounding) lines in Fig. 1. Thus, for any given rotation, the moment capacity value is assumed to lie in a closed interval, the extreme values of which represent upper (strongest) and lower (weakest) bound responses. Such intervals can also take into account the uncertainties associated with manufacturing defects, construction errors, etc. that are invariably present [18].

The approach used is a significant extension of our recent interval elastoplastic analysis work [19] to incorporate the effects of semi-rigid connection properties. The present paper focuses on the formulations and solution approaches for an interval analysis belonging to the class of sophisticated nonlinear mechanics problems. In essence, it considers the simultaneous presence of intervals associated with initial stiffnesses and strain-hardening (non-perfect plasticity) material properties that describe the uncertain properties of semi-rigid connections fabricated in practical steel structures.

Following the general approach known as a "convex" model [16], the semi-rigid joint constitutive law is first described in an interval format, after suitable piecewise linear (PWL) discretization [20] of upper and lower bounding moment-rotation curves. Such computationally (and in fact theoretically also) advantageous PWL discretizations were popularized by Maier [21] in the 1970s in the context of mathematical programming approaches to elastoplastic analyses. Detailed expressions for the PWL elastoplastic hardening laws for both noninterval and interval assumptions are derived in Section 2. This allows the development, in Section 3, of suitable beam finite elements with a semi-rigid connection at either or both ends. In Section 4, a path-independent (or holonomic) assumption, which has been shown to be sufficiently accurate for predicting nonholonomic (path-dependent) structural responses under a monotonic load regime [22], leads to an interval

elastoplastic analysis formulation as a nonstandard mathematical programming problem known as an interval mixed complementarity problem (or interval MCP). Interval uncertainties in the applied load can also be included. The interval MCP admits an infinite number of solutions (viz. corresponding to all possible moment-rotation values within the intervals adopted), and is thus neither solvable nor necessary to obtain all solutions. Since we aim to obtain extreme structural responses for each applied load level, we convert (Section 5) the interval MCP to a pair of nonstandard optimization problems, so-called interval mathematical programs with equilibrium constraints (or interval MPECs) [19]. For each load level, we can solve these two interval MPECs to provide the most favorable response in one case, and the least favorable in the other. In Section 6, we describe a simple and efficient reformulation using a smoothing regularization technique [23] to solve the interval MPECs. Three examples are provided in Section 6 to illustrate the approach and to highlight the influence of semi-rigid connection variability on structural response.

#### 2. Semi-rigid connection

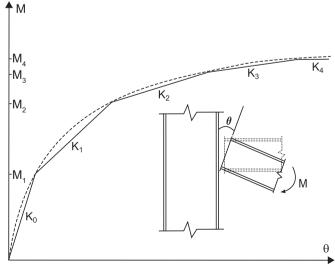
#### 2.1. Noninterval description

The flexural (in-plane) behavior of steel beam-to-column connections is described by the interface moment M and corresponding relative rotation  $\theta$  measured from the column surface. A typical moment-rotation curve in Fig. 2 (dashed line) typifies the nonlinear hardening behavior of standard semi-rigid joints. We approximate the actual nonlinear curve by an appropriate number of yield hyperplanes [21,3,20].

For the PWL yield diagram in Fig. 2 of a generic semi-rigid connection at a beam end a, let  $K_0$  denote the initial tangent stiffness;  $K_1$  to  $K_4$  gradients associated with the four hardening branches 1–4, respectively; and  $M_1$  to  $M_4$  moments corresponding to the break points. For a perfectly plastic final stage,  $M_4$  would be the maximum moment capacity, with  $K_4 = 0$ .

The rotation  $\theta$  is the additivity of the relative elastic rotation  $\theta_e$  and its plastic counterpart  $\theta_p$  (viz.  $\theta = \theta_e + \theta_p$ ). We decompose, in Fig. 3, the  $M-\theta$  law into its elastic and plastic components;  $\theta_e$  and  $\theta_p$  are respectively elastic and plastic rotations, and  $h_1$  to  $h_4$  hardening parameters.

We adopt the classical Koiter's non-interactive hardening rule [24]. This hypothesis is reasonable as elastic unloading is unlikely



**Fig. 2.**  $M - \theta$  response of generic semi-rigid connection.

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