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Numerical analysis of large elasto-plastic deflection of constant curvature beam under follower load

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ABSTRACT

This paper describes a method to analyze the elasto-plastic large deflection of a curved beam subjected to a tip concentrated follower load. The load is made to act at an arbitrary inclination with the tip tangent. A moment-curvature based constitutive law is obtained from linearly hardening model. The deflection governing equation obtained is highly non-linear owing to both kinematics and material non-linearity. It is linearized to obtain the incremental differential equation. This in turn is solved using the classical Runge–Kutta 4^{th} order explicit solver, thereby avoiding iterations. Elastic results are validated with published literature and the new results pertaining to elasto-plastic cases are presented in suitable non-dimensional form. The load to end angle response of the structure is studied by varying normalized material and kinematic parameters. It is found that the response curves overlap at small deflection corresponding to elastic deformation and diverge for difference in plastic property. The divergent response curves intersect with each other at higher deflection. The results presented also show that the approach may be used to obtain desired non-uniformly curved beam by suitably loading a uniform curvature beam.

1. Introduction

Large deflection of slender elastic beam under the influence of a terminal force a.k.a elastica has been of interest since the time of Euler [1]. With the invention of smart materials, engineering applications involving large beam deflection have expanded considerably over the last few decades [2]. Many of such applications involve curved beams with terminally acting follower forces inducing inelastic large deflections. In the present work an attempt is made to introduce a simple approach to predict and subsequently study these problems.

The elastica problem in its simplest form is that of a horizontal slender cantilever under the action of a conservative vertical terminal load. The large deflection in elastica is essentially a quasi-static phenomenon due to finite rotation under small strain condition applied to an Euler–Bernoulli beam. Assuming this condition to hold true, the elastica problem has evolved to include material non-linearity, non uniform cross section, and non-conservative forces etc.

The governing equation of the elastica is a two point boundary value problem and is non-linear due to geometric non-linearity. Analytical solutions to such problems are limited to evaluation of elliptic or Jacobian integrals. For a recent review on analytical methods, the reader may find [3] useful.

The analytical solutions are implicit in nature for load or displacement and are generally expressed in terms of the end slope. To obtain explicit load–displacement responses required in engineering applications, various semi-analytical techniques are devised. Homotopy perturbation and Adomian decomposition methods are two of the most popular semi-analytical techniques; some of the relevant ones are [4–8] and many of the references therein. The semi analytical techniques generally deal with simpler variations of elastica dealing with elastic problems and render solutions in the form of long expressions.

To solve more complicated problems of elastica like non-linear elasticity, various numerical approaches are adopted. Though FEM appears to be the most popularly adopted approach, many complicated elastica problems can be solved by easier or more computationally economic approaches. These non-FEM approaches may be broadly looked into as an algorithm which involve a numerical integration scheme coupled with a root finding iterative technique [9,10]. Yu and Johnson [11] coined the terminology 'plastica' indicating an extension of the closed form elastica theory to incorporate plasticity. They solved the problem a cantilever under conservative compressive force using the perturbation technique coupled with numerical integration, considering an elastic-perfectly plastic materials model. Refs. [12,13] considered bi-linear elasto-plastic moment–curvature based constitutive law

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in solving large deflection problems of slender structures.

Non-conservative terminal force which follow the deformed beam profile, induce additional complexity for analysis as it poses the question of adequacy of static approaches. It was Beck [14] who first estimated the buckling load of an elastic column under a tangentially acting compressive follower load by dynamic analysis. Rao and Rao [15] found that static approaches are applicable to sub-tangential follower load systems, provided the load is lesser than the critical flutter (dynamic instability) causing force.

Though an extensive literature exist for problems of elastic straight beam under follower forces [16–19], relatively less literature deals with curved beam under follower forces. In the seminal work of Argyris and Symeonidis [20], elastic curved beam under various follower loads is studied in depth by employing FEM. Spric and Saje [21] used finite difference method to study the large deflection of curved elastica under tip concentrated follower load. Nallathambi et al. [22] solved the curved cantilever beam under a tip concentrated follower load by numerical integration starting from free end coupled with one parameter shooting method. Shvartstman [23] solved the same by direct integration of an initial value problem obtained by change of variable from the two point boundary value problem. Additionally he showed that a curved elastic beam under follower load can become unstable only by flutter i.e. to say divergence instability does not exist. Very recently, Ghuku and Saha [24] obtained closed form solution of a curved cantilever elastica under dead load. So far, only planar cases have been discussed. In a recent work by [25], a comprehensive analysis has been carried out on both in-plane and out-of-plane response using an intrinsic formulation and shooting.

The motivation of the present work is driven by a relative dearth of literature dealing in large elasto-plastic deflection of curved beam under follower load. Evidently additional complexity in terms of material non-linearity is brought in by considering plasticity into the existing geometrically non-linear problem. The aim of the paper is to present a simple explicit methodology of solving such problems and analyze the numerical results. Such problems when studied in detail may find application in fields like lumbar spine prosthetic, smart structures, flexible robotic arms, and nonuniform curvature hook manufacturing. The methodology adopted here consists of linearizing the governing nonlinear equation about current time to obtain an incremental form of the differential equation for the current step which in turn is solved by employing Runge–Kutta 4th order method. The problem is solved using the static method hence the current “time” actually refers to pseudo time instants.

The mathematical formulation for a general elasto-plastic curved beam is presented in Section 2. In Section 3 the solution methodology is explained. In Section 4, results pertaining to various curves and loading are presented and discussed. An example application of the results is also presented herein.

2. Formulation

A constant curvature cantilever of length l and included angle γ completely describes its geometry in un-deformed configuration, as shown in Fig. 1. With tip follower load P oriented at an angle α with respect to the tangent at its tip deforms the beam to produce an end angle of ψ with horizontal. Clearly, in the un-deformed configuration, $\psi=\gamma$ when $P=0$. The assumptions, scope and the equation of the curved cantilever are described in detail in this section. The development of the incremental constitutive law used in the present analysis is also presented in this section.

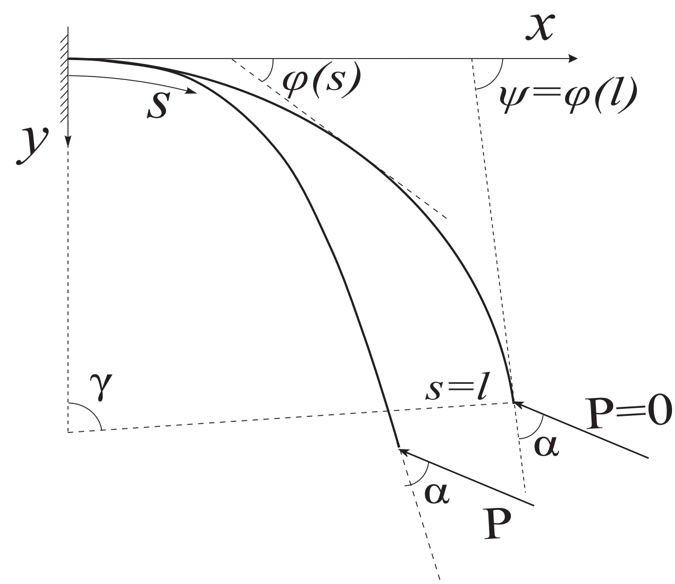

Fig. 1. Schematic of a general curved cantilever under a general follower force at its tip.

2.1. Assumption and scope

The beam is assumed to follow Euler–Bernoulli hypothesis and hence axial and shear deformations are neglected. It is assumed to be of uniform curvature in the un-deformed state. It is made up of homogeneous isotropic elastic-linearly hardening rate independent material. A quasi static load acts in the plane of symmetry of the beam following the deformation of the beam maintaining a constant angle (α) with the tip tangent (see Fig. 1). The beam is assumed to undergo large curvature change within small strain framework. In this work, static analysis methodology is adopted, hence the solutions are limited to statically stable configurations.

2.2. Governing relations

In Fig. 1, the un-deformed configuration is shown in which a Cartesian coordinate system XOY is chosen wherein OX axis is directed along the right horizontal while OY points vertically down. The arc length is measured from the fixed end along the deformed beam and is denoted by s. The tangent at any point on the beam axis makes an angle $\phi(s)$ with OX axis. For convenience in notation, the end angle is denoted by ψ i.e. $\phi(s = s_{max} = l) = \psi$. The load P is assumed to follow the beam such that it maintains α orientation with respect to the beam tangent at its tip.

To derive the governing equation of deformation from statics, we note from kinematics, the curvature is given by:

$$\kappa(s, t) = \frac{\partial \phi}{\partial s} \tag{1}$$

Considering kinetics at any s along the deformed beam, and differentiating the hogging bending moment M with respect to s, we get:

$$\frac{\partial M}{\partial s} = -P \sin(\alpha - \psi + \phi) \tag{2}$$

A general rate independent material model relating the moment and curvature, may be conveniently expressed as:

$$D = \frac{dM}{d\kappa} \tag{3}$$

In Eq. (3), D denotes the elasto-plastic flexural rigidity of the beam.