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A simple nonlinear model to simulate the localized necking and neck propagation

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ABSTRACT

This paper deals with the equilibrium problem in nonlinear dissipative inelasticity of damaged bodies subject to uniaxial loading and its main purpose is to show the interesting potentialities offered by the damage theory in modeling the necking and neck propagation phenomena in polymeric materials. In detail, the proposed mechanical model is a two-phase system, with the same constitutive law but with different levels of damage for each phase. Despite its simplicity, it is shown that the model can straightforwardly reproduce the overall load–elongation curve provided by experimental tensile tests by involving only five parameters of clear physical meaning.

1. Introduction

Many semi-crystalline and glassy polymeric materials exhibit non-homogeneous inelastic large deformations under uniaxial tension, with deformation localizations leading to necks of reduced cross-sectional area. At appropriate conditions, the neck boundaries may propagate along the specimen. This process, termed cold drawing, is nowadays the basis of much film and fiber processing.

Considère published the basic criterion for necking in 1885 [1]. In 1932, Carothers and Hill [2] observed that necks in filaments of semi-crystalline polyester propagate near the room temperature. Later, Whitney and Andrews [3], Crissman and Zapas [4], Zapas and Crissman [5] found that under certain conditions necking could also occur in glassy polymers.

Orowan [6] and Nadai [7] provided a first mechanical explanation of cold drawing in terms of homogeneous elongations. Barenblatt [8] suggested a theory of polymer necking, proposing a one-dimensional approach based on a special stress-diffusion phenomenon, theoretically necessary for stabilizing the neck propagation. However, this assumption did find no experimental basis.

To describe the cold drawing in terms of nonlinear elasticity, Antman [9,10] understood the importance of non-uniform deformations in necking. In spirit of the approximate theory for long elastic bars, Antman, using an averaging procedure and defining an energy functional, reduced the problem to the search of minimizers of this functional and to the study of stability and bifurcation conditions.

Ericksen [11] considered a non-monotonic constitutive law, with a serpentine shape, and demonstrated a remarkable similarity between the necking and phase transition. He established conditions for the coexistence of necked and unnecked states in a bar subject to uniaxial tension and, using the above similarity, he proved that the neck propagation can occur at the constant Maxwell stress, since this value makes energetically equal both phases. However, this model is not able to describe the formation of a localized necking and also requires a particular form of the constitutive law for the bar to justify the neck propagation.

Coleman [12] incorporated in the theory of nonlinear elasticity a one-dimensional inhomogeneity in the necking region. In particular, he proposed a special constitutive relation for the dependence of the tension in a thin polymeric fiber on the variation of the stretch along the fiber axis. With this theoretical assumption, Coleman computed equilibrium solutions which are able to describe necks, bulges, drawing configurations and periodic striations. However, looking at the experimental evidences, it is not clear if a specimen under uniform tension can experience a so complex variation of the longitudinal stretch.

Necking and neck propagation were analyzed numerically by

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several authors. Needleman [13] and Burke and Nix [14] studied the hypothetical plasticity effects in necking. Hutchison and Neale [15] proposed a three-dimensional analysis using the J_2 flow theory for plasticity to characterize the inelastic behavior of polymers.

Tugcu and Neale [16,17] carried out a finite-element analysis of necking under axisymmetric and plane strain conditions. The results presented in these analyses were the computed overall load–elongation response of the bar, as well as the evolution of the specimen profile and the stress distribution in the bar at various stages of the deformation process.

The numerical solution by Silling [18] of a two-dimensional elastic necking problem, with a non-monotonic constitutive law, demonstrated the closeness between the Maxwell and calculated actual neck propagation stress.

The necking has been intensively investigated experimentally. The model proposed by Peterlin and Olf [19] is the most popular. They considered the folded chain blocks in necking of semi-crystalline polymers as tilted, sheared, broken off the lamellae, which are incorporated in the amorphous microfibrils.

Gent and Jeong [20] and Gent and Madan [21] have more recently proposed another model for semi-crystalline polymers. They related necking to the mechanism of unfolding chains in crystalline blocks and transferring them into the amorphous phase with consequent orientation. The necking is so explained as a mechanical melting of the folded chain blocks.

Experimental aspects of the constitutive behavior of a polycarbonate (Lexan) have been investigated accurately in the work by Buisson and Ravi-Chandar [22], where a grid technique has been used to determinate the non-homogeneous deformation field associated with a steadily growing neck. However, in these experimental works there has been no attempt to derive a mechanical constitutive relation to describe the necking.

In all the above-mentioned theoretical, numerical and experimental works there was a fundamental lack in proposing and developing mechanical models, based on the experimental evidences, in order to describe thoroughly the necking and neck propagation phenomena. In the general framework of continuum mechanics, a new modeling of necking and neck propagation problems is presented in this paper by using the theory of damage.

The theory of damage is particularly suitable for studying the inelastic effects in polymeric materials [23–25]. In the context of infinitesimal theory, the damage mechanics was introduced, about fifty years ago, by Kachanov [26] and then developed by Chaboche [27,28], Lemaitre [29–31] and Krajcinovic [32].

A generalization to large deformations has been proposed by Simo [33] and Simo and Ju [34]. Subsequently, several authors developed phenomenological models with damage to describe the Mullin effect [35–41]. The damage in amorphous materials has been studied by Horgan et al. [42] and by De Tommasi et al. [43,44].

Using concepts from irreversible thermodynamics, some contributions on the time evolution of damage for polymeric materials have been achieved by Rajagopal et al. [45]. With the so-called weak formulation, some analytical studies have been performed by Mielke [46,47]. In detail, he defined an energy functional and a dissipation potential and sought solutions satisfying both a condition of global stability and an energy balance.

Recently, the equilibrium problem in nonlinear dissipative inelasticity of damaged bodies subjected to uniaxial loading has been treated in the paper by Lanzoni and Tarantino [48]. In this work, applying the continuum thermodynamics theory, the constitutive law for damaged materials and an inequality for the energy release rate are derived. After having formulated the equilibrium boundary-value problem, explicit expressions governing the global development of the equilibrium paths are computed.

The equilibrium solutions obtained in [48] will be extensively applied in this paper. Thus, the main results achieved in [48] will be briefly recalled in the next section. In Section 3, some experimental overall load–elongation curves for high-density polyethylene (HDPE) are shown, and a qualitative description of the sequence of events unfolding in an uniaxial test is provided. The reference model is composed of an undamaged phase and by a damaged phase, where the necking occurs. This model is presented in Section 4, where the equilibrium conditions are imposed and the main energetic aspects are discussed. After having modeled a localized necking, the results are shown through a series of diagrams. In Section 5, a purely energetic criterion is proposed to evaluate the fundamental values of a neck propagation under steady-state condition. Using the modeling carried out, the paper closes reproducing the overall load–elongation curve describing the localized necking and neck propagation phenomena.

2. Preliminaries and basic equations

The nonlinear damage formulation recently proposed by Lanzoni and Tarantino [48] will be used in this work to study isothermal and rate-independent[1] equilibrium problems of damaged bodies under uniaxial tractions. In [48], a damaged material is defined as a material characterized by a reduced capacity to store energy (other contributions can be found in [52,53]).[2] Thus, the effects of damage have been described by a proper damage function that has been directly included in the variable list of the internal energy. With such an assumption, the constitutive law for damaged hyperelastic materials and an inequality for the energy release rate have been derived in the framework of the continuum thermodynamics. Subsequently, the equilibrium boundary-value problem for bodies composed of damaged isotropic materials has been formulated and explicit expressions governing the global development of the equilibrium paths have been obtained. Along these paths, the damage evolves and accumulates according to the density energy level reached during the deformation process.

In [48], equilibrium solutions for a body $\mathcal{B}$ having the shape of a rectangular parallelepiped are presented. A sketch of the body $\mathcal{B}$ is depicted in Fig. 1, where the reference system adopted is also referred. Body forces are disregarded and it is supposed that the body is stretched and maintained in equilibrium under the sole action of surface forces s, applied normally and uniformly on the two basis. These are considered dead loads, namely, applied forces characterized by a density per unit area in the reference configuration which does not depend on the deformation. The body $\mathcal{B}$ is composed of a compressible neo-Hookean material, whose undamaged and damaged stored energy functions, denoted by w_0 and w, are expressed in the following form:

$$w_0 = \hat{w}_0\left(\lambda_1,\ \lambda_2,\ \lambda_3\right) = a\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2\right) + c\left(\lambda_1^2\,\lambda_2^2\,\lambda_3^2\right) - q\ \ln\left(\lambda_1\,\lambda_2\,\lambda_3\right) - \left(3a + c\right), \tag{1}$$

$$w = (1 - d)w_0, \tag{2}$$

with

$$d = \hat{d}(w_0) = d_\infty\left[1 - \exp\left(-\frac{w_0 - \widetilde{w}_0}{\eta}\right)\right], \quad \text{for } w_0 \geq \widetilde{w}_0, \tag{3}$$

[1] Examples of time-dependent mechanical systems can be found in [49–51].

[2] The stored energy function of an undamaged material in correspondence of the apex of a crack is unbounded (see, e.g., [54–58]). On the other hand, it is bounded for a Bell material [59].