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A numerical assessment of the load bearing capacity of externally pressurized moderately thick tubes

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A B S T R A C T

The collapse behavior of cylindrical shells pressurized from outside is examined. Attention is focused on tubes of moderate thickness, as required by very deep water pipelines or some innovative nuclear power plant proposals. Their collapse is expected to be dominated by yielding but, because of the decreasing nature of the post-collapse evolution, interaction with instability is likely to be significant enough to demand consideration. At present, no quantitative assessment of such effect is available, because little study has been devoted to tubes in this thickness range.

Plasticity–instability interaction is activated by imperfections and to assess their influence on a systematic numerical study is undertaken. Computations produce a meaningful measure of the collapse pressure and it is proposed that the allowable pressure be determined on its basis, by introducing a suitable safety factor. This is chosen so that results reproduce those provided by presently accepted procedures in the well explored and reliable range of medium-thin tubes. When the same factor is applied to thicker tubes, the resulting allowable pressure is significantly higher than the values suggested by codes, which apparently react to the present lack of knowledge by assuming an extremely conservative attitude.

1. Introduction

Buckling of shells under external pressure is a satisfactorily settled subject when shells are thin enough to collapse in the elastic range. In some situations, however, shells of higher thickness are required: medium-thin cylinders are presently employed in oil industry as pipes or casings and, since pipe laying in increasingly deeper water is envisaged, increasingly thicker tubes become of interest; also, recent proposals for innovative nuclear power plant design consider tubes of moderate thickness pressurized from outside [1].

Medium-thin tubes, typical of oil industry applications, are considered in a number of papers [2–9], mostly providing numerical computations of the collapse pressure. Attempts at reproducing results with empirical design formulas are also made [2,3,9] and the approximations obtained are adequate in the thickness range explored. However, such formulas, often borrowed from the only partially similar problem of beam-columns, contribute marginally to the understanding of the behavior and their use can be justified only on numerical ground.

This makes at least questionable their direct application to thicker tubes, whose collapse behavior has been little explored both from the experimental and the numerical points of view. When employed in this range existing design formulas turn out to be extremely conservative and accepted design procedures, such as those based on *ASME Boiler & Pressure Vessel* code [10], produce surprisingly high values for the thickness, reflecting a substantial lack of knowledge on the phenomena involved. This situation has significant consequences for the IRIS (*International Reactor Innovative and Secure*) project. In its design, steam generator tubes are contained inside the vessel and pressurized from outside [1]. Present ASME Section III rules require an external diameter to thickness ratio less than 8.5: this entails a major contribution of the thermal conduction through the wall thickness to the thermal resistance in the heat exchange process between primary and secondary fluids, with detrimental consequences on the dimensioning of the heat transfer surface.

It is felt that ASME code requirements are exceedingly conservative in this thickness range. Reason is that imperfections are not explicitly considered when defining a reference failure pressure. Provided that their entities are below given values, imperfections are accounted for by means of a safety factor that is essentially slenderness independent. However, at different slenderness imperfections have different effects on the load bearing capacity. The most detrimental ones are experienced when the interaction

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between plasticity and instability is strongest, which occurs for medium-thin tubes, and the safety factor required by the most severe situation is unnecessarily high in other instances.

This paper aims at a precise assessment of the collapse behavior of long and possibly thick cylindrical shells. A systematic numerical study is undertaken to this purpose, consisting of large displacement, elastic–plastic analyses up to collapse, explicitly accounting for imperfections. It is shown that the effects of imperfections of different nature exhibit the same dependence on slenderness and that, among all of them, initial out of roundness ("*ovality*") most significantly affects the pressure bearing capacity of the tube and can be taken as representative of all effects of this kind.

The results obtained are thought to be useful under at least three respects. First, they provide an insight on the collapse behavior of tubes in a thickness range so far overlooked. Secondly, they permit the identification of the imperfections that most significantly affect the overall strength and that must be included in a numerical model when complete non-linear analyses are required. In addition, they can be used for preliminary design purposes: the computed collapse pressure can be taken as the *reference failure pressure* and the *allowable pressure* can be defined by applying a suitable safety factor to it. It is proposed that this factor be chosen so as to reproduce ASME sizing for medium-thin tubes, a range explored sufficiently well to ensure that codes consider the proper safety margin. The same factor can be applied to tubes of any slenderness, including comparatively stocky ones.

2. Theoretical limit values

A cylindrical shell of nominal circular shape, with outer diameter D and wall thickness t, is subjected to external pressure q. The material is isotropic, elastic-perfectly plastic and governed by von Mises' criterion. E and ν are its elastic constants (Young modulus and Poisson ratio, respectively) and σ_0 denotes its tensile yield strength.

The shell is long enough for end effects to be disregarded. In this situation, it can reasonably be assumed that axial strains are uniform, even if different from zero. If axial stresses vanish on the average, the limit pressure of the theoretical perfect tube is given by the smaller of the following values:

$$\text{Elastic buckling pressure} \quad q_E = 2\frac{E}{1-\nu^2}\frac{1}{\frac{D}{t}\left(\frac{D}{t}-1\right)^2} \tag{1a}$$

$$\text{Plastic limit pressure} \quad q_0 = 2\sigma_0\frac{t}{D}\left(1+0.5\frac{t}{D}\right) \tag{1b}$$

The first expression is well known (see Ref. [11]) while equation (1b) is, for any $D/t > 6$, an excellent approximation to the exact value established in Ref. [12] (results in [12] apply to thicker tubes as well, but they are not reproduced by equation (1b), a simple modification to the thin shell solution). A very similar formula, with coefficient 0.5 replaced by 0.47, was proposed in Ref. [3] on empirical basis.

Equations (1) are adequate for moderately thick cylinders ($8 \le D/t \le 15$), the case of prominent interest in this paper. Even if not as thick as those used in high pressure technology, such tubes are thick enough for stress variation over their thickness to be considered. Nonetheless, the average value S of the hoop stress is a meaningful piece of information. Its value is dictated by equilibrium and reads

$$S = \frac{1}{2}q\frac{D}{t} \tag{2}$$

Since peak stresses may exceed this value by a significant amount, equation (2) is only an alternative (and often convenient) way to refer to pressure. In particular, the theoretical limits (1) may be replaced by the expressions:

$$S_E = \frac{E}{1-\nu^2}\frac{1}{\left(\frac{D}{t}-1\right)^2} \quad S_0 = \sigma_0\left(1+\frac{1}{2}\frac{t}{D}\right) \tag{3}$$

which are obtained by substituting in equation (2) either of the values of equations (1) for q.

3. Tube sizing according to ASME requirements

Nuclear class 1 components must obey the requirements of Section III (Division 1, Subsection NB) of the *ASME Boiler & Pressure Vessel* code [10]. As an alternative, Code Case 2286-1 [13] can be used. This was originally intended at replacing specific Section VIII requirements, but its Section III counterpart was recently approved as Code Case N-759 [14].

Table 1 summarizes the results provided by the two sizing procedures for the specific case of the IRIS steam generator tube bundles, which are made of nickel–chromium–iron alloy UNS N06690 (*INCONEL 690*) and operate at the design temperature of $T = 345\,°C$. With respect to ASME Section III, Code Case 2286-1 sizing allows for a 43% thickness reduction, with the D/t ratio increasing from less than 8.3 to nearly 13. This suggests that Section III requirements entail significant conservativeness.

Fig. 1 compares the allowable working pressures predicted for long tubes of different D/t ratios, made of *INCONEL 690* at $T \approx 350\,°C$. Results are expressed in terms of the average hoop stress, as defined by equation (2), which, being less sensitive than pressure to D/t variations, makes the picture easier to understand. The *theoretical failure* values, the smaller among the Euler buckling and the plastic collapse loads, equation (3), are also shown (gray curve). For their computation the following mechanical properties are assumed

$$E = 183\ \text{GPa}, \quad \sigma_0 = 240\ \text{MPa} \tag{4a}$$

while the Poisson ratio is taken as

$$\nu = 0.289 \tag{4b}$$

Equation (4a) is consistent with the chart that provides, for the material and the temperature under consideration, the value of *factor B*, on which ASME sizing is based. Equation (4b) replaces the material independent value $\nu \approx 0.3$ that ASME assumes in all cases. The difference has negligible effects on results.

An alternative representation is given in Fig. 2, depicting the ratios of the theoretical failure pressure to the allowable values predicted by the two procedures, i.e. the safety factors of the two rules effectively assumed with respect to the ultimate load of the hypothetical perfect tube. Comparison shows that Code Case requirements are, sometimes significantly, less severe than Section III rules. For $D/t > 30$ differences are only in the safety factor: the two codes give the same interpretation to failure, even if their

Table 1
IRIS SG tube bundle design according to ASME requirements – Inner diameter $D_{int} = 13.24$ mm, design pressure $p_d = 17.24$ MPa.

		ASME III	CC 2286-1
Design thickness	t (mm)	2.11	1.21
Design outer diameter	D (mm)	17.46	15.66
	D/t	8.27	12.94
Allowable pressure	p_a (MPa)	17.62	17.67
	p_a/p_d	1.022	1.025