International Journal of Non-Linear Mechanics 74 (2015) 51–72

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

On the non-classical mathematical models of coupled problems of thermo-elasticity for multi-layer shallow shells with initial imperfections

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ARTICLE INFO

Article history:
Received 11 December 2014
Received in revised form
4 March 2015
Accepted 25 March 2015
Available online 3 April 2015

Keywords:
Non-classical theory
Shallow multi-layer shells
General solutions
Non-linear boundary value problems

ABSTRACT

Mathematical modeling of evolutionary states of non-homogeneous multi-layer shallow shells with orthotropic initial imperfections belongs to one of the most important and necessary steps while constructing numerous technical devices, as well as aviation and ship structural members. In first part of the paper fundamental hypotheses are formulated which allow us to derive Hamilton–Ostrogradsky equations. The latter yield equations governing shell behavior within the applied hypotheses and modified Pelekh–Sheremetev conditions. The aim of second part of the paper is to formulate fundamental hypotheses in order to construct coupled boundary problems of thermo-elasticity which are used in non-classical mathematical models for multi-layer shallow shells with initial imperfections. In addition, a coupled problem for multi-layer shell taking into account a 3D heat transfer equation is formulated. Third part of the paper introduces necessary phase spaces for the second boundary value problem for evolutionary equations, defining the coupled problem of thermo-elasticity for a simply supported shallow shell. The theorem regarding uniqueness of the mentioned boundary value problem is proved. It is also proved that the approximate solution regarding the second boundary value problem defining condition for the thermo-mechanical evolution for rectangular shallow homogeneous and isotropic shells can be found using the Bubnov–Galerkin method.

1. Introduction

In general, investigation of either static or dynamic states of real shells with the help of mathematical modeling is reduced to the following fundamental steps: (i) construction of mathematical models within the proposed hypotheses; (ii) proofs of a formal agreement of the models considered; (iii) numerical tests of the models introduced; (iv) analysis of the numerical results and their validation via laboratory experiments.

Using both the 3D Hamilton–Ostrogradsky variational equation and the 3D heat-transfer equation, the novel geometrically non-linear boundary value problems are formulated which yield (in the frame of the non-classical theory of shells) the evolutionary states of thermo-elastic one- and multi-layer shallow shells taking into account clamping of the layers and initial imperfections.

For the case of non-classical one-layer shells with clamping a cubic approximation with regard to thickness of the longitudinal displacements is applied, and a linear approximation with respect to transversal displacement is used. In the case of a non-classical model of multi-layer shells without clamping, a cubic approximation along thickness of longitudinally displacements is used, and the independent behavior of layers transversal displacement versus thickness is taken into account. Besides, in order to reduce the number of governing functions, the Pelekh–Sheremetev conditions are applied and their modifications for orthotropic shells of variable thickness are introduced.

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http://dx.doi.org/10.1016/j.ijnonlinmec.2015.03.011
0020-7462/

A brief historical description of the studied problem follows. In the middle of the 20th century a substantial development of the theory of mechanics of solids embedded in various physical fields is observed, which originated from the technical requirements of aviation, cosmic, electronic and nuclear industries [1–15]. The achieved level of investigations regarding coupled and non-coupled problems of thermo-elasticity has been reported in monographs [16–26].

It should be noticed that for the coupled problems of thermo-elasticity of plates and shells there are no papers aiming at estimation of the limits of application of classical and mathematically improved models. The fundamental results regarding the existence and uniqueness of solution of linear coupled problems of 3D thermo-elasticity were obtained on the basis of the method of potentials in the theory of multidimensional singular integral equations in references [27,28]. The mentioned works include classical solutions of both internal and external problems of thermo-elasticity. In particular, the problem of existence of solutions in the defined boundary value problems is addressed. In general, the mentioned problems were also studied by mathematicians (see, for instance, references [29–38]).

Consequently, the occurrence of the mentioned unsolved problems has a negative influence on the quality and reliability of analysis of the numerical results obtained while investigating the evolutionary states and dynamical stability loss of the deformed constructions and machine devices. Namely, in order to achieve full and reliable states of a given continuous thermo-elastic mechanical system (shell) one must apply the appropriate phase or configuration spaces, but their choice should be certainly motivated by theorems of existence of solutions for the boundary problems governing the used models of shells. This is implied by the Newton–Laplace principle of determinism.

Recently, great interest of engineers in manufacturing large-scale one-shot structures using straight-fiber laminate composite materials has been observed. The problem of designing guidelines for taper sections of composite laminates applied in the aerospace industry is still not satisfactorily solved. Beams, plates and shells made from functionally graded material (FGM) produced from a mixture of two different materials (metal and ceramics) play nowadays a key role in engineering and industrial branches. The composite structural members are usually optimized with respect to reduction of their weight; they often work in a thermal environment. Owing to complexity of the problem in view of modeling and computation, there is a need for development of a mathematical theory for non-classical models of designing non-homogeneous shallow shells, which include proper descriptions of the corresponding phase spaces yielding novel mathematical methods devoted to modeling of evolutionary states of shallow shells. They should be supported by the norms of phase spaces and the dedicated algorithms to carry out reliable and validated computational experiments.

The paper is organized in the following way. First, in Section 2 the boundary value problem of a 3D shell is formulated. Next, assuming a multi-layer thermo-elastic shell composed of orthotropic layers of variable thickness, a 3D heat transfer equation is added and the interaction between temperature and deformation fields is formulated. It allows us to define the coupled boundary problems of thermo-elasticity. The phase space and second boundary value problem are studied in Section 4 (an important theorem is formulated and proved). Computational examples of the kinetic models of flexible Euler–Bernoulli, Timoshenko and Pelekh–Sheremetev beams are illustrated and discussed in Section 5. Section 6 summarizes the results putting emphasis on their novelty.

2. Boundary value problems

Let a shell treated as a 3D (see Fig. 1) deformed body take in its non-deformed state (in space R^3) a bounded measurable space D with boundary surface ∂D, $\overline{D} = D \cup \partial D$ – closure of space D ($\mu\, D \neq 0$, where $\mu(\cdot)$ – volume of space D).

Here and further, the following conditions are applied:

(1) We take from D a smooth surface, called further the reference surface of the investigated shell, and we apply its first square form defined within the Euclidean metric in space R^2;
(2) Space R^3 is parameterized with the help of rectangular coordinates $Ox_1x_2x_3$, where co-ordinates x_1, x_2 correspond to the lines of curvature of the reference surface, and axis Ox_3 coincides with a normal to the reference surface and points into the center of the curvature;
(3) Vectors $\overline{e}_j$ ($j = 1, \ldots, 3$) create an orthonormal basis for the given system of coordinates;
(4) The reference surface is defined by $x_3 = 0$;
(5) Spaces D and $\overline{D}$ are straight cylinders

$$D = \Omega \times (\delta_0,\ \delta_n),\ \overline{D} = \overline{\Omega} \times [\delta_0,\ \delta_n],\ \overline{\Omega} = \Omega \cup \partial\Omega$$

where $\overline{\Omega}$ stands for the orthogonal projection of $\overline{D}$ onto the reference surface, Ω is the opened space with introduced Lebesque metrics, and $\partial\Omega$ is its contour; $\delta_0 = \delta_0(x_1,\ x_2)$, $\delta_n = \delta_n(x_1,\ x_2)$ are the known functions defined on $\overline{\Omega}$ and describing bases of cylinders D and $\overline{D}$;
(6) Function $h = \delta_n(x_1,\ x_2) - \delta_0(x_1,\ x_2)$, $(x_1,\ x_2) \in \overline{\Omega}$ defines variable thickness of the shell in point $(x_1,\ x_2)$, and we have

$$\forall\ (x_1,\ x_2) \in \overline{\Omega},\ h > 0,\ \delta_0 \neq 0,\ \delta_n \neq 0$$

(7) We use time interval $[t_0,\ t_1]$ regarding time variable t.

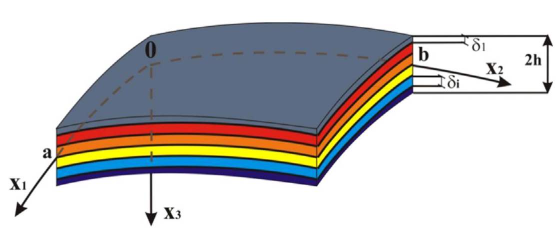

Fig. 1. A multi-layer shell model (the used notation is given in the text).