International Journal of Non-Linear Mechanics 74 (2015) 92–99

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Stochastic non-linear response of a flexible rotor with local non-linearities

J.-J. Sinou [a,*], J. Didier [a], B. Faverjon [b]

[a] Laboratoire de Tribologie et Dynamique des Systèmes UMR 5513, Ecole Centrale de Lyon, France
[b] Laboratoire de Mécanique des Contacts et des Structures UMR 5259, INSA Lyon, France

ARTICLE INFO

Article history:
Received 27 November 2014
Received in revised form
31 March 2015
Accepted 31 March 2015
Available online 8 April 2015

Keywords:
Non-linear vibration
Uncertainty
Rotor
Harmonic balance method
Polynomial chaos

ABSTRACT

The effects of uncertainties on the non-linear dynamics response remain misunderstood and most of the classical stochastic methods used in the linear case fail to deal with a non-linear problem. So we propose to take into account of uncertainties into non-linear models, by coupling the Harmonic Balance Method (HBM) and the Polynomial Chaos Expansion (PCE). The proposed method called the Stochastic Harmonic Balance Method (Stochastic-HBM) is based on a new formulation of the non-linear dynamic problem in which not only the approximated non-linear responses but also the non-linear forces and the excitation pulsation are considered as stochastic parameters. Expansions on the PCE basis are performed by passing via an Alternate Frequency Time method with Probabilistic Collocation (AFTPC) for estimating the stochastic non-linear forces in the stochastic domain and the frequency domain. In the present paper, the Stochastic Harmonic Balance Method (Stochastic-HBM) that is applied to a flexible non-linear rotor system, with random parameters modeled as random fields, is presented. The Stochastic-HBM combined with an Alternate Frequency-Time method with Probabilistic Collocation (AFTPC) allows us to solve dynamical problems with non-regular non-linearities in presence of uncertainties. In this study, the procedure is developed for the estimation of stochastic non-linear responses of the rotor system with different regular and non-regular non-linearities. The finite element rotor system is composed of a shaft with two disks and two flexible bearing supports where the non-linearities are due to a radial clearance or a cubic stiffness. A numerical analysis is performed to analyze the effect of uncertainties on the non-linear behavior of this rotor system by using the Stochastic-HBM. Furthermore, the results are compared with those obtained by applying a classical Monte-Carlo simulation to demonstrate the efficiency of the proposed methodology.

1. Introduction

During the past decade, numerous studies have been done to understand and to model the non-linear phenomenon structural dynamics. Most of these models deal with determinist parameters. But in the design of many types of engineering systems, geometrical, materials and non-linear parameters are often uncertain due to the manufacturing process for example. So an expected improvement and an obvious extension of the classical deterministic studies are the consideration of uncertainties when designing a system according to certain operating conditions. However, the effects of uncertainties on the non-linear dynamics response of mechanical systems remain misunderstood and most of the classical stochastic methods used in the linear case fail to deal with a non-linear problem. This is especially true in the context of designing rotating machines [1–3].

Numerous methods have been developed to take into account uncertainties in the dynamic response: the classical perturbation methods, the Monte Carlo method or the Polynomial Chaos Expansion for example. The perturbation methods are based on the development of random quantities in Taylor or Neumann series [4–6]. These methods have proved their effectiveness for small variations. One of the most classical methods is Monte Carlo Simulations (MCS) which include uncertainties by generating samples of the random parameter in the deterministic model . This method can be used to deal with a linear or a non-linear problem but MCS have a high computational cost due to the fact that a high number of samples are necessary to reach the converged response of the mechanical system under study. To reduce the computational cost, one of the most useful methods is the Polynomial Chaos Expansion (PCE) [7,8]. This method has proved its robustness and efficiency on linear dynamic problems (and more specifically for analyzing the dynamics of rotating machines [9–12]) but PCE does not seem adapted to deal with multiple solutions observed in non-linear systems.

* Corresponding author. Tel.: +33 472186466.
E-mail address: jean-jacques.sinou@ec-lyon.fr (J.-J. Sinou).

http://dx.doi.org/10.1016/j.ijnonlinmec.2015.03.012

So we propose to take into account uncertainties into non-linear rotor models by applying a novel non-linear method called the Stochastic Harmonic Balance Method (Stochastic-HBM [13]). This approach is an extension of the Harmonic Balance Method (HBM) that is one of the mathematical approaches to solve deterministic equations with non-linear terms [14–19] and the Polynomial Chaos Expansion (PCE) that is one of the most used stochastic parametric methods [7,8]. In addition an Alternating Frequency Time method [20] with Probabilistic Collocation (PC) for calculating the stochastic non-linear forces in stochastic domain and frequency domain is proposed. This approach proposes also a new formulation of the non-linear stochastic dynamic problem in which the excitation pulsation is considered as a stochastic parameter and expanded on the PCE basis. Numerical simulations for a simple non-linear two-degree-of-freedom model with different types of non-linearities were carried out to appraise the global methodology. Considering uncertainties in linear and non-linear parts of the mechanical system, it was demonstrated that the quasi-periodic stochastic dynamic response evaluated by this approach agreed very well with those obtained from the classical Monte Carlo Simulation. Moreover, it is found that the results obtained via the Stochastic-HBM require significantly less computation.

So the objective of the present study is to demonstrate efficiencies and strong capabilities of this numerical approach to calculate an accurate stochastic response of large rotating systems and to discuss the need to consider both non-linear terms and uncertainties in rotating machines in order to obtain an accurate solution in a design process.

Table 1
Value of the physical parameters.

Description	Value
Radius of the rotor shaft	0.02 m
Length of the rotor shaft	1 m
Position of disk 1	0.6 m
Outer radius of the disk 1	0.1 m
Thickness of the disk 1	0.02 m
Position of disk 2	0.8 m
Outer radius of the disk 2	0.2 m
Thickness of the disk 2	0.02 m
Young's modulus of elasticity	2.1×10^{11} N m^2
Shear modulus	8×10^{10} N m^2
Density	7800 kg m^{-3}
Poisson ratio	0.3
Stiffness of supports	10^6 N m^{-1}
Mass unbalance	0.0005 kg
Clearance value r_{lim}	0.0001 m
Non-linear stiffness k_{nl}	10^{11} N m$^{-1/3}$
Stiffness k_2	10^6 N m^{-1}

The paper is organized as follows: firstly, the non-linear rotor system with uncertainty is presented. Then, the process to calculate the dynamic response of the stochastic rotor model is presented by detailing the Stochastic-HBM combined with the Alternate Frequency-Time method and the Probabilistic Collocation. Finally, numerical simulations are proposed in order to illustrate the main capabilities of the proposed methodology and to investigate the influence of both non-linearities and uncertainties for a flexible rotor system.

2. Rotor system with uncertainties

2.1. General equation of the rotor system

The rotor system under study consists of a shaft with two discs. The shaft is discretized into 10 Euler beam finite elements with four degrees of freedom at each node (two lateral displacements and two rotations, the axial and torsional degrees of freedom not being considered). Two bearing supports are added at each end of the rotor. All the material properties and dimensions of the rotor are given in Table 1. The layout of the rotor system is shown in Fig. 1(a) and the nodal displacement vector defined as $[v_1\ w_1\ \theta_1\ \psi_1\ v_2\ w_2\ \theta_2\ \psi_2]$ in fixed frame is given in Fig. 1(b).

After assembling the different elements of the rotor [21,3], we obtain the equation of motion of this rotor system:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+(\mathbf{C}+\omega\mathbf{G})\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t)+\mathbf{f_{nl}}(\mathbf{x}(t)) \quad (1)$$

with **M**, **K** and **G** being the mass, stiffness and gyroscopic matrices of the complete system (the shaft, the two rigid disc and the two bearing supports), respectively. Matrix expressions can be found in [9]. **C** defines the matrix of Rayleigh damping of the shaft. ω is the rotational speed of the shaft. **f** corresponds to the linear forces (i.e. gravitational forces and unbalance forces) applied to the rotor system. $\mathbf{f_{nl}}$ contains the non-linear forces. The non-linearity is located only on the first bearing. Two kinds of non-linear forces have been conducted in order to demonstrate the efficiency and robustness of the proposed approach:

- *A regular cubic polynomial non-linearity*: The restoring force at the first bearing support for both the x-direction and y-direction are given by

$$\begin{cases} f_{nl_x}(t) = -k_{nl}v^3(t) \\ f_{nl_y}(t) = -k_{nl}w^3(t) \end{cases} \quad (2)$$

where k_{nl} defines the additional non-linear term. v and w are the relative displacements in the x-direction and y-direction, respectively, as illustrated in Fig. 1(b)

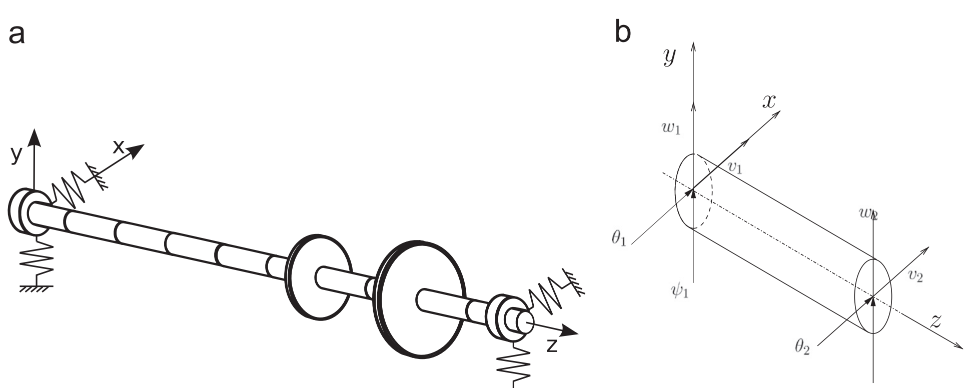

Fig. 1. Finite-element model of the rotor system: (a) rotor system and (b) coordinates in fixed frame.