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Water hammer simulation by implicit method of characteristic

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A R T I C L E I N F O

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A B S T R A C T

An Implicit Method of Characteristics is proposed in this paper to alleviate the shortcomings and limitations of the mostly used conventional Method of Characteristics (MOC). An element-wise definition is used for all the devices that may be used in a pipeline system and the corresponding equations are derived in an element-wise manner. The proper equations defining the behavior of each device including pipes are derived and assembled to form the final system of equations to be solved for the unknown nodal heads and flows. Proposed method allows for any arbitrary combination of devices in the pipeline system. The method is used to solve two example problems of transient flow caused by closure of a valve and failure of a pump system and the results are presented and compared with those of the explicit MOC. The results show the ability of the proposed method to accurately predict the variations of head and flow in all cases considered.

1. Introduction

Water hammer is produced by a rapid change of flow velocity in the pipelines that may be caused by sudden valve opening or closure, starting or stopping the pumps, mechanical failure of a device, rapid changes in demand condition, etc. It could result in violent change of the pressure head, which is then propagated in the pipeline in the form of a fast pressure wave leading to severe damages. The velocity of this wave may exceed 1000 m/s and the values of pressure may oscillate from very high to very low values. Design and operation of any pipeline system requires that the distribution of head and flow in the system is predicted at different operating conditions. Many researchers have attempted the simulation of transient flow in pipeline systems with different methods.

Chaudhry and Hussaini [1] solved water hammer equations by MacCormak, Lambda and Gabutti explicit Finite Difference (FD) schemes. They found that these second-order FD schemes result in better solutions than the first-order MOC. Izquierdo and Iglesias [5] developed a computer program to simulate hydraulic transients in a simple pipeline system by mathematical modeling [5]. They also presented the users with a powerful tool to plan the potential risks to which an installation may be exposed and to develop suitable protection strategies. Their model produced good numerical results within the accuracy of the used data. This model was later generalized to include a pumping station fitted with check valve, delivery valve and two air vessels [8]. Ghidaoui et al. [6] proposed a two- and five-layer eddy viscosity model for water hammer simulation. A dimensionless parameter, i.e. the ratio of the time scale of the radial diffusion of shear to the time scale of wave propagation, was proposed to estimate the accuracy of the assumption of flow axisymmetry in the water hammer phenomenon. Filion and Karney [7] proposed a method that combined a numerical integration method with a transient simulation model to improve the accuracy and capabilities of extended-period simulations in pipe networks. This method analyzes a water distribution system for short time periods near the start and end of a time step using a transient model, and then uses a modified Euler's method to predict the behavior of the system. Their method leads to a significant increase in simulation accuracy, but requires more system information and computational effort. Zhao and Ghidaoui [9] formulated first- and second-order explicit finite volume (FV) methods of Godunov-type [9] for water hammer problems. They compared the performances of FV schemes and MOC schemes with space line interpolation for three test cases with and without friction. They modeled the wall friction using the formula of Brunone et al. [17]. It was found that the first-order FV Godunov-scheme produces the same results with MOC using space line interpolation. It was also shown that, for a given level of accuracy, the second-order Godunov-type scheme requires much less memory storage and execution time than the first-order Godunov-type scheme. Recently, Kodura and Weinerowska [11] investigated the difficulties that may arise in modeling of water hammer phenomenon. Sibetheros et al. [3] showed that in numerical analysis of water hammer in a frictionless horizontal pipe, the performance of method of characteristics (MOC) could be considerably improved by interpolations using spline polynomials. Wood [10,13] compared MOC and Wave characteristics Method (WCM) showing that for the same modeling accuracy, the WCM

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requires less execution time. In addition, he showed that the number of calculations per time step required by WCM does not increase when more accuracy is required while for the MOC, the number of calculations per time step is roughly proportional to the accuracy. Ghidaoui and Kolyshkin [4] performed linear stability analysis of the base flow velocity profiles for laminar and turbulent water hammer flows. They found that the main parameters that govern the stability of the transient flows are the Reynolds number and a dimensionless time scale. Saikia and Sarma [12] presented a numerical model using MOC and Barr's explicit friction factor [14] for solution of the water hammer problems. The proposed model was examined for rapid valve closure in downstream of a long conduit with a reservoir upstream. The stability and accuracy of the method was tested by comparing the results with those of the Lax Diffusive Method [2]. Greyvenstein [16] proposed an implicit finite difference method based on the simultaneous pressure correction approach. The method could be indiscriminately used for both liquid and gas flows based on isothermal and non-isothermal behaviors.

In what follows, the explicit method of characteristic is first described along with the boundary conditions required for reservoirs, valves and pumps in Section 2. In Section 3, the basic concepts of proposed implicit method of characteristics (IMOC) is first described for pipe elements and is then extended to derive the equations governing reservoir, valve and pump elements. The application of the proposed method to two benchmark examples from the literature is described in Section 4. And finally the paper is concluded with the conclusions in Section 5.

2. Conventional explicit MOC

Analyses of the most hydraulic transients are based on one-dimensional continuity and momentum equations. Various numerical approaches have been introduced for the simulation of the pipeline transients. These include the Method of Characteristics (MOC), Wave Characteristics Method (WCM), Finite Volume Method (FVM), Finite Element Method (FEM) and Finite Difference Method (FDM). Among these methods, MOC is the mostly used method for its simplicity and superior performance over other methods.

In the MOC, the convection terms are omitted from the governing differential equations to arrive at the following relations:

$$\frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial x} + RQ|Q| = 0 \tag{1}$$

$$a^2\frac{\partial Q}{\partial x} + gA\frac{\partial H}{\partial t} = 0 \tag{2}$$

where $R = f/(2DA)$ = term of friction factor, Q = discharge, H = pressure head, A = area of the pipe, a = velocity of the pressure wave, g = acceleration due to gravity, t = time, f = friction factor of the pipe, D = diameter of the pipe, and x = distance along the pipe. The MOC approach transforms the water hammer partial differential equations into the ordinary differential equations along the characteristic lines defined as

$$C^+: \quad \frac{dQ}{dt} + C_a\frac{dH}{dt} + RQ|Q| = 0 \quad \frac{dx}{dt} = a \tag{3}$$

$$C^-: \quad \frac{dQ}{dt} - C_a\frac{dH}{dt} + RQ|Q| = 0 \quad \frac{dx}{dt} = -a \tag{4}$$

where $C_a = gA/a$.

Integrating these equations on the characteristic lines between time steps t and $t+\Delta t$, shown in Fig. 1, and solving them for the known variables lead to

$$C^+: \quad Q_P = C_P - C_aH_P \quad C_P = Q_A + C_aH_A - R\Delta tQ_A|Q_A| \tag{5}$$

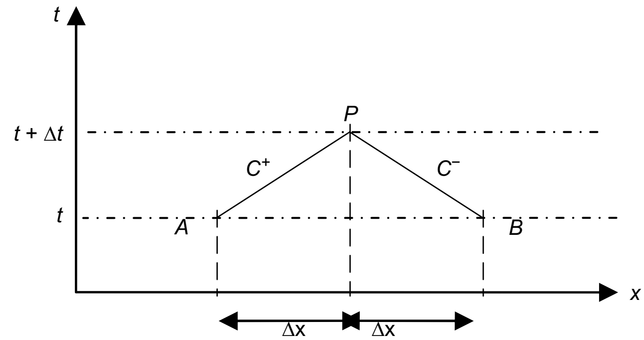

Fig. 1. Characteristic lines in x–t plan.

$$C^-: \quad Q_P = C_n + C_aH_P \quad C_n = Q_B - C_aH_B - R\Delta tQ_B|Q_B| \tag{6}$$

in which Q_P = unknown flow at point P at time $t+\Delta t$, H_P = unknown head at point P at time $t+\Delta t$, Q_A, Q_B = flows at neighboring sections of P at the previous time t, and H_A, H_B = heads at neighboring sections of P at the previous time t. Discretising each pipe of the pipeline system into segments of length Δx, the above equations can be used to calculate the pressure head and flow at all the interior points of the pipes. Calculation of flow variables at two end-points of each pipe segment requires appropriate boundary conditions depending on the type of device used in the system.

2.1. Boundary conditions

There may be different types of devices such as check valves, junctions, pressure-reducing valves, surge tanks, pumps, etc., located between pipes in a pipeline system. In the MOC, these apparatuses are usually treated as the boundary conditions for equations governing the transient flow in pipes. For this, the characteristic equations are combined with the proper boundary conditions to arrive at the final equations for the boundary nodes. This, however, means that only one device can be used between any two pipes, which is a serious limitation if the simulation of a general pipeline system is required. In what follows the boundary conditions imposed by the most common devices in pipeline systems are briefly addressed. A comprehensive discussion of these boundary conditions can be found elsewhere [15].

2.1.1. Reservoir

Reservoir is one of the most important devices used in the pipeline systems. The main characteristic of a reservoir, shown in Fig. 2, is that the water level in the reservoir remains constant during the transient conditions. For a reservoir at the upstream end of a pipeline system, the following equation holds [15]:

$$H_{P_{i,1}} = H_{\rm res} - (1+k)\frac{Q^2_{P_{i,1}}}{2gA^2} \tag{7}$$

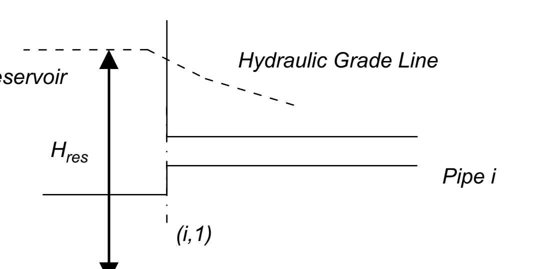

Fig. 2. Constant level upstream reservoir.