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Development of new analytical solutions for elastic thermal stress components in a hollow cylinder under sinusoidal transient thermal loading

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ABSTRACT

For the analysis of high-cycle thermal fatigue due to striping (such as has been observed due to turbulence at mixing tees of class 1–2–3 piping of nuclear power reactors) it can be necessary to consider the time-dependent temperature gradient within the pipe wall thickness rather than just at the surface. To address this, a set of analytical solutions with several new features has been developed for the temperature field and the associated elastic thermal stress distributions for a hollow circular cylinder subjected to sinusoidal transient thermal loading at the inner surface. The approach uses a finite Hankel transform and some properties of Bessel functions. The analytical predictions have been successfully benchmarked by comparison with results from finite element analysis, and also with some results of independent studies.

1. Introduction

The development of thermal fatigue damage due to turbulent mixing or vortices in light water reactor (LWR) piping systems is still not fully understood [1,2]. In particular, accurate representation of the load is a complex issue and much effort continues to be devoted to experimental and analytical studies in this area [3,4]. As opposed to the relatively low number of cycles associated with thermal stratification, thermal striping[2] at vortices and in mixing areas is more high cyclic in nature [5].

Thermal gradients and turbulence in the coolant fluid can induce oscillating local stresses in the portion of the pipe near the inside surface if the flow rates are sufficiently high. Numerical simulations of thermal striping often assume a sinusoidal fluctuation of the fluid temperature; results show that the oscillation frequency of the temperature of the coolant is a key factor in the response of pipe wall temperature field and that the critical frequency range is 0.1–1 Hz [6]. The amplitude of the metal temperature oscillations is smaller than the difference in the hot and cold coolant layers because of the finite value of the heat transfer coefficient and the thermal inertia of the pipe. The high-cycle fatigue damage caused by thermal stresses is initially limited to the pipe inner surface adjacent to the interface, and further crack growth depends on the thermal stress profile through the thickness of the pipe [7–11]. The current study addresses this issue by the development of the analytical solutions for the through-wall temperature response and associated thermal stress components in a hollow cylinder due to sinusoidal transient thermal loading.

The new solution scheme can be used to support several elements of a proposed European Thermal Fatigue Procedure [2] for high-cycle fatigue damage assessment of mixing tees, including load spectrum analysis based on one-dimension temperature and stress evaluations at each measured location and also thermal fatigue crack growth assessment.

2. Short background on methods applied to solving thermoelasticity problems for thermal transients

Thermal stresses are defined [12,13], as self-balancing stresses produced by a non-uniform distribution of temperature or by differing thermal coefficients of expansion. These thermal stresses are developed in a solid body whenever any part is prevented from assuming the size and shape that it would freely assume under a change in temperature. Localized thermal stresses are associated with almost complete suppression of the differential expansion and thus no significant distortion of the overall body. Such stresses are considered only from the fatigue

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[2] Thermal striping is defined as the effect of a rapid random oscillation of the surface temperature inducing a corresponding fluctuation of surface stresses and strains in the adjacent metal. It is characterized by large numbers of strain cycles having potential to add to any fatigue damage produced by strain cycles associated with other plant operation transients.

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Nomenclature

$r_i = a$	inner radius of the pipe
$r_o = b$	outer radii of the pipe
θ	temperature change from the reference temperature
T_0	reference temperature
r	radial distance
κ	thermal diffusivity
λ	thermal conductivity
ρ	mass density
c	specific heat coefficient
$q(t)$	function of time representing the thermal boundary condition
$J_\upsilon(z)$, $Y_\upsilon(z)$	Bessel functions of first and second kind of order υ
θ_0	amplitude temperature wave
ω	wave frequency in rad/s
t	time variable
s_n	positive roots of the transcendental equation
u	radial displacement
ν	Poisson's ratio
α	coefficient of linear thermal expansion
E	Young's modulus
ε_{zz}	axial strain
σ_{rr}	radial stress
$\sigma_{\theta\theta}$	hoop stress
σ_{zz}	axial stress
σ_y	yield stress
L, **M**, **N**	operators
T	finite integral transform
H	finite Hankel transform
$K(x, \xi)$	kernel of integral transform

standpoint and are therefore classified as local stresses. Examples include the stress at a small hot spot on a vessel wall and the difference between the actual stress and equivalent linear stress resulting from a radial temperature distribution in a cylindrical shell. In looking at the methods for analyzing thermal stress under thermal transients, we shortly mention those published for hollow cylinders (pipes). Segal [14] has studied the transient response of a thick-walled pipe subjected to a generalized excitation of temperature on the internal surface using Duhamel's relationship. The generalization of the temperature excitation was achieved using a polynomial composed of integral-and half-order terms. In Ref. [15] Lee, Kim and Yoo applied a numerical approach using Green's function method (GFM) for analysis of crack propagation under thermal transient loads. They have shown that GFM can be efficiently used to evaluate thermal stresses for fatigue analysis and also for stress intensity factors (SIFs) for crack propagation analysis. The same authors reported [16] an evaluation procedure for thermal striping damage on secondary piping of liquid metal fast reactors (LMFR) using GFM and standard finite element method (FEM). Shahani and Nabavi [17] solved the quasi-static thermoelasticity problem in a thick-walled cylinder using the finite Hankel transform for the differential equations of both temperature and displacements. Marie [18] proposed an extension of the analytical solution for the temperature and stresses in the event of a linear shock in a pipe containing a fluid by a simple solution for any variation of the temperature in the fluid. The approach consists of breaking down the fluid temperature variation into a succession of linear shocks. Noda and Kim [19] used a Green's function approach based on the laminate theory to solve the two-dimensional unsteady temperature field and associated thermal stresses in an infinite hollow circular cylinder. The unsteady heat conduction equation was formulated as an eigenvalue problem by making use of eigenfunction expansion and laminate theory. The associated thermoelastic field was analyzed using the thermoelastic displacement potential function and Michell's function.

In the present work we were specifically interested in an analytic formulation which could be applied to a wide range or pipe geometries and temperature conditions relevant to coolant piping systems. None of the above approaches were available found to be fully suitable: in some cases the geometry boundary conditions were inappropriate, in others the published information was insufficient to allow direct implementation. As a result, it was decided to develop a new solution to meet our requirements; this is presented in the following sections.

3. Temperature distribution in a hollow cylinder subjected to sinusoidal transient thermal loading

The calculation of the temperature distribution in a piping subsystem must be distinguished from that in components with more complex geometries. Pipes can be represented by a hollow cylinder and with such a simple geometry it becomes possible to use analytical tools to get the time-dependent temperature profile through wall thickness. Hence a pipe wall model subject to a sinusoidal fluctuation of fluid temperature can be used for assessment of thermal stripping damage phenomenon. A suitable analytical solution of time-dependence temperature in pipes provides a basis for obtaining solutions for the associated thermal stress components and their profile through thickness. This approach facilitates extraction of stress intensity ranges for computing cumulative usage factors (CUFs) and of stress profiles for crack growth analysis.

The Laplace, Fourier, Hankel and Mellin transforms have been applied to the solution of boundary-value problems in mathematical physics [20–22]. The application of such transforms reduces a partial differential equation in n independent variables to one in $n-1$ variables and it is often possible, by successive operations of this type, to reduce the problem to the solution of an ordinary differential equation. In applying the method of integral transforms to problems formulated in finite domains it is necessary to introduce finite intervals on the transform integral. Transforms of this nature are called finite transforms. Sneddon [21] considered a Bessel function as a kernel of a finite integral which he defined as a Hankel transform and showed its usefulness for solving certain boundary-value problems. The Hankel transform arises naturally in problems posed in cylindrical coordinates which are solved using the technique of separation of variables, involving Bessel functions [21,23].

Let us consider an infinite hollow cylinder of the inner and outer radii $r_i = a$ and $r_o = b$, respectively. Also, assume that the cylinder is made of a homogeneous isotropic material. The one-dimensional heat diffusion equation in cylindrical coordinates is [24,25]:

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} = \frac{1}{\kappa}\frac{\partial\theta}{\partial t}, \quad (a \le r \le b, \ \ t \ge 0) \tag{1}$$

where

$$\theta(r,t) = T(r,t) - T_0, \tag{2}$$

is the temperature change from the reference temperature at any radial position r and at time t. The reference temperature T_0 is the temperature of the body in the unstrained state or the ambient temperature before changing of temperature. Also,