



The Gent model for rubber-like materials: An appraisal for an ingenious and simple idea



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ABSTRACT

We review the main aspects of the celebrated Gent constitutive model for rubberlike materials. Emphasis is placed on the case of damageable materials describing possible damage and deformation localization.

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1. Introduction

During 1996 Alan Gent publishes a short note [16] where he proposed a new constitutive equation for the non-linear elastic behavior of rubberlike materials. Due to its formal simplicity, this model reached a great popularity in various scientific communities interested in large elastic deformations of solids. The aim of this note is to present an appraisal of this simple constitutive model by showing its effectiveness in describing the behavior of traditional and many new elastomeric and biological materials. We remark that the Gent model [16] has been applied successfully in several different fields of material science and in the following we refer only to a small subset of the many possible applications of the simple idea behind the Gent model. For a more detailed survey of the scientific literature related to the Gent model, we refer the reader to the paper by Horgan in this same volume [18].

Rubberlike materials are typically characterized by their ability to undergo very large reversible deformations. As a consequence they are modeled (at least in appropriate ranges of stretches, temperature and time scales) as hyperelastic materials. A general treatment of such an approach was given by Rivlin [32] and later by many other authors (see, for example, [25]). This peculiar macroscopic behavior arises from the structure of rubber-like materials at the micro- (network) scale and we refer to the classical book by Treloar [34] for an analysis of the amorphous properties of such macromolecular materials at these scales.

To be more explicit, let us consider a deformation $\hat{\mathbf{x}}$ of a body Ω , $\Omega \ni \mathbf{X} \mapsto \mathbf{x} = \hat{\mathbf{x}}(\mathbf{X})$ and let $\mathbf{F} = \text{Grad}(\hat{\mathbf{x}})$ be the deformation

gradient. Let then $\mathbf{B} = \mathbf{F}\mathbf{F}^T$ be the left Cauchy–Green strain tensor and let

$$I_1 = \text{tr}(\mathbf{B}), \quad I_2 = \frac{I_1^2 - \text{tr}(\mathbf{B}^2)}{2}, \quad I_3 = \det \mathbf{B}$$

be its principal invariants. If a material is hyperelastic and isotropic [25] we may introduce a strain-energy density function $W = W(I_1, I_2, I_3)$. Moreover, because the bulk modulus of rubber is typically significantly higher than the tensile modulus, rubber and rubber-like materials are often modeled as incompressible materials with an energy density depending on only the first two invariants: $W = W(I_1, I_2)$.

As a result, the Cauchy stress tensor \mathbf{T} is given by

$$\mathbf{T} = -p\mathbf{I} + 2\frac{\partial W}{\partial I_1}\mathbf{B} - 2\frac{\partial W}{\partial I_2}\mathbf{B}^{-1}, \quad (1.1)$$

where p is the Lagrange multiplier associated with the incompressibility constraint. Moreover, in the absence of body force, the equilibrium equations are

$$\text{div } \mathbf{T} = \mathbf{0} \quad (1.2)$$

(here div is the usual divergence operator with respect to \mathbf{x}).

Schematically, we may consider three distinct classes of constitutive approaches proposed in the framework of non-linear elasticity for rubberlike materials. The first class proposed phenomenological constitutive laws, relating them both to the experimental observations and to the known molecular structure of the material. In this class different simple microstructure based constitutive equations were taken into account to describe the thermo-mechanical behavior of rubber-like materials. The most representative strain energy density function in this class is for sure

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the basic neo-Hookean model

$$W = \frac{E}{6} (I_1 - 3), \quad (1.3)$$

depending on one single material small strain tensile modulus E that has been related in various ways to the molecular material properties. This macroscopic constitutive law results as a *homogenized* behavior of an isotropic network of ideal Gaussian chains. An introduction to this class of models can be found in the classical work of Treloar [34].

Successfully the axiomatic theory of Continuum Mechanics [35] delivered a systematization of the so-called *representation problem* by a systematic use of the methods of Linear Algebra (see for example [32]). Thus, the second class of constitutive models focused on the *mathematical* determination of specific forms of the general representation formula (1.1). Several new interesting models resulted from the research in this direction among which the most widely adopted is the Mooney–Rivlin model considering an energy function depending linearly also on the second invariant

$$W = C_1(I_1 - 3) + C_2(I_2 - 3). \quad (1.4)$$

Here C_1 and C_2 are two material moduli. Of course (1.3) is a special case of (1.4).

The third, more recent, class of constitutive theories resulted from a critical reassessment of all previous theories searching for a deeper connection between the macroscopic response of rubber-like materials and the behavior at the mesoscopic scale. The aim of these new models was both to cover the gap between the theoretical predictions and the experimental behavior and to generalize the elastic constitutive theories to more general material behaviors (e.g. residual strains and Mullin and Payne effects). In this case the main contribution was the introduction of models based on the inverse Langevin function defined as $\mathcal{L}(x) = \coth(x) - 1/x$ [3].

To understand the role of this function, we briefly recall that rubberlike materials are amorphous materials consisting of very long flexible chainlike molecules constituted by a backbone of many non-collinear single valence bonds around which, due to thermal agitation, barrier free rotations are possible. Rubbery materials are composed of a network of thousands of these chains linked through interchain bonds. These topological links deliver the material a *solid behavior* while allowing the molecules to change their shape easily and continuously at normal temperatures due to their Brownian motion. If a tensile force is applied to the network then the chains assume a somewhat oriented configuration. Several interesting material behaviors descend from this microscale material structure. For example, if the macromolecules in the network are highly regular they are able to finely pack (for stereochemical reasons) so that they easily crystallize due to van der Waals forces. In this case the polymeric material behaves like a crystalline solid exhibiting ductile and plastic behavior. In other cases the monomers may hardly move due to viscous, inertial and entanglement effects under thermal agitation and therefore the polymer behaves in a *glassy* way.

The ability of the Gent model to describe important properties of rubberlike materials can be substantially noted by the following observation of the behavior of these materials at the network scale. As described above, rubber materials are constituted by long chains energetically free to switch between a wide variety of conformations through transitions' phenomena which are governed mainly by the statistics of random processes. For this reason we say that the elasticity of rubber-like materials is *mainly* entropic in nature, because their elasticity is regulated by the configurational entropy of the chains and not by enthalpic energy contributions. In particular, the statistics that governs the conformation of the macromolecules over short chain lengths is

Gaussian, but, when we approach something like one-third of the fully stretched length of the chain (*contour length*), non-Gaussian effects become non-negligible. Macroscopically this is reflected in a large experimental discrepancy of the predictions of a model such as (1.3) based on the Gaussian assumption, or its phenomenological generalization (1.4), with experimental observations.

To overcome this problem, two main methods have been proposed. In one direction more complex phenomenological forms than (1.4) have been proposed. Often the main drawback of these constitutive models is that they contain a large number of empirical parameters that are not always easily determined by fitting procedures with experimental data. The advantage is that they are ready to use in the commercial finite element codes as greatly appreciated by engineers.

The second, successful, direction of research was dedicated to the refinement of the molecular description of the macromolecular network and to the consideration of the non-Gaussian character of the random processes of the chains conformations, taking into account that the probability density function for the end-to-end distance of the chains has compact support, and not infinite tails as in the Gaussian case, since the macromolecules have finite contour lengths. These studies generated several models, among which the most popular is the Arruda–Boyce eight chain model [2] which is indeed based on the inverse Langevin function. The advantage of such models is that they usually contain few constitutive parameters and that such parameters are directly connected to the microscopic properties of the material. On the other hand the main disadvantage is the numerical and analytical complexity of such models and the fact that the stress–strain relationship is usually not expressed in closed explicit form.

The Gent constitutive equation has to be considered in such framework (Paraphrasing Gent himself [16])

a simple, two-constant, constitutive relation, applicable over the entire range of strains. ...

Alan Gent, using the typical British understatement, justifies the introduction of his model only on the basis of the simple mathematical feasibility. In reality, it was an ingenious idea that allows, especially because of its mathematical simplicity, exploration of many mechanical issues associated with the elasticity of rubber-like materials. These aspects are reviewed in detail in the paper by Horgan in this issue [18] and we refer to this paper for a more detailed survey of the various results obtained in the literature about the Gent model [18]. The aim of this paper is to show the main features (and limits) of the Gent model by considering the simple deformation classes of simple shear and rectilinear shear. Then we show how the idea of Gent led the derivation of a new generation of models useful also in the description of important biomaterials like spider silk or protein macromolecular biomaterials.

2. The Gent model

The energy density function proposed by Gent in [16] for incompressible, isotropic, hyperelastic materials is, using his exact formulation,

$$W = -\frac{E}{6} J_m \ln \left[1 - \frac{I_1}{J_m} \right]. \quad (2.1)$$

Here E is the small strain tensile modulus that, for incompressible materials, is related to the infinitesimal shear modulus μ by the relation $\mu = E/3$. Thus, since W depends on the only first invariant of \mathbf{B} , the Gent model belongs to the class of the *generalized* neo-Hookean materials such that $W = W(I_1)$.

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