International Journal of Non-Linear Mechanics 68 (2015) 132–145

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

A simple model for myocardial changes in a failing heart

John A. Shaw [a,*], Kannan Dasharathi [b], Alan S. Wineman [c], Ming-Sing Si [d]

[a] Department of Aerospace Engineering, University of Michigan, 1320 Beal Ave, Ann Arbor, MI 48109, USA
[b] Department of Aerospace Engineering, University of Michigan, Ann Arbor, MI 48109, USA
[c] Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109, USA
[d] Department of Cardiac Surgery, University of Michigan, 1540 E. Hospital Dr. SPC 4204, Ann Arbor, MI 48109, USA

ARTICLE INFO

Article history:
Received 19 April 2014
Received in revised form
25 June 2014
Accepted 25 June 2014
Available online 1 August 2014

Keywords:
Biomechanics
Heart failure
Multi-network hyperelasticity

ABSTRACT

In a simplified setting, a multi-network model for remodeling in the left ventricle (LV) is developed that can mimic various pathologies of the heart. The model is an extension of the simple model introduced by Nardinocchi and Teresi [9], Nardinocchi et al. [10,11] that results in an algebraic relation for LV pressure–volume–contraction. We considered two networks, the original tissue and a new tissue, each of which has its own volume fraction, stress-free reference configuration, elastic properties, and contractility. This is used to explore the consequences of microstructural changes in the muscle tissue on LV function in terms of the pressure–volume loop during a single cardiac cycle. Special attention is paid to the stroke volume, which is directly related to cardiac output, and changes in LV wall stress caused by various disease states, including wall thinning (dilated cardiomyopathy), wall thickening (hypertrophic cardiomyopathy), contractility degradation, and stiffness changes (scarring). Various scenarios are considered that are of clinical relevance, and the extent and nature of remodeling that could lead to heart failure are identified.

1. Introduction

Heart failure is a medical condition with significant individual and societal impact [1,2]. Approximately 1 in 9 deaths in the USA are associated with congestive heart failure (CHF) and 50% of adults diagnosed with CHF will die in 5 years [1–3]. Further, up to 5% of neonates born with congenital heart disease have single ventricle heart disease with a deficient ventricular mass that cannot support a biventricular repair and undergo surgical palliation within the first 2 years of life [4–6]. This palliation results in a decreased life expectancy, a severely limited exercise capacity and significant predisposition to end-stage CHF later in life [7]. Therefore gaining further understanding of the mechanics of heart failure may have significant implications for the guidance and design of treatment strategies. Although there is better understanding of cardiac disease at the molecular and cellular levels, there remains a definite need for simple mathematical models at the organ level to aid physiologists and clinicians. Many different approaches and models used to analyze heart failure and ventricular mechanics have been proposed since the 1970s, yet even a basic pressure–volume loop analysis of the heart has not yet been standardized [8].

Our aim is to describe and model the mechanical behavior of the left ventricle (LV) in the setting of certain cardiomyopathies and to explore the conditions under which ventricular function degrades to critical levels. The recent mathematical model of the heart by Nardinocchi and Teresi [9], Nardinocchi et al. [10,11] (henceforth termed the NTV model) is appealing in its simplicity to capture the essential features of the pressure–volume cycle of the LV. The goal of the NTV is mathematical tractability, while ignoring many well known complicating and individualized aspects of actual physiology. Our extension to the NTV model investigates the global implications of remodeling in the myocardium that may result in changes in LV function and cardiac output. More precise and complex models of LV function exist [12–14]; however, they all require detailed numerical methods and large scale computations.

The NTV model decomposes muscle deformation into two parts [9]: (1) a change of reference configuration due to electrochemically induced contraction (active deformation), and (2) the purely elastic deformation due to applied (external) loads from this new reference configuration (mechanical deformation). This interpretation is entirely consistent with the multi-network framework for materials undergoing microstructural changes introduced by Wineman and Rajagopal [15,16], in which a constitutive

* Corresponding author. Tel.: +1 734 764 3395; fax: +1 734 764 0578.
E-mail addresses: jashaw@umich.edu (J.A. Shaw), kannand@umich.edu (K. Dasharathi), lardan@umich.edu (A.S. Wineman), mingsing@umich.edu (M.-S. Si).

http://dx.doi.org/10.1016/j.ijnonlinmec.2014.06.015
0020-7462/

theory was developed for elastomers undergoing scission and crosslinking of their macromolecular structure due to large deformation and elevated temperatures. Further extension to account for dissolution and re-assembly of macromolecular networks in natural rubber was carried out by Shaw et al. [17]. The consequences of macromolecular structure changes on the mechanical response were explored in a number of studies on elastomers [18–23], fiber reinforced composites with elastic matrix and fibers [24], and a study of the mechanical interaction between a contracting muscle and a deformable body [25].

This notion of active deformation formed the basis of the NTV model for the study of myocardial contractions [10,11]. In particular, the LV was modeled as a thin-walled spherical container composed of homogeneous muscle tissue undergoing uniform circumferential stretch. The actual stretch ratio in the muscle tissue was the product of a mechanical stretch ratio and one representing muscle contraction. When the muscle contracted, the radius of the spherical container attempted to decrease, but encountered resistance by the contained blood. This led to mechanical stretch and stress in the container wall and pressure on the blood. The muscle tissue forming the container was described as a non-linear elastic material for which the stress was given in terms of a mechanical stretch ratio. With this approach, it was possible to discuss ventricular pressure–volume–contractility response under various conditions without the complications arising from the specific architecture of the extracellular matrix and muscle fibers as well as their individual constitutive properties.

This special issue is dedicated to the memory of our colleague Alan Gent, who was a master at distilling complex problems to their essential features and providing an insightful, simplified analysis. We offer our simple model of LV function in the same spirit. The NTV model provides a convenient and intuitive means to introduce and explore additional modeling ideas without the complications arising from heterogeneous physiological, spatial, and biomechanics details. The particular idea of interest here is that of microstructural change of muscle tissue which is analogous to scission and crosslinking of a deforming elastomeric network structure.

Inspired by the NTV model, the purpose of the present work is to introduce microstructural changes and to study their consequences. The microstructural change could be due to excessively large deformations or non-mechanical effects arising from various disease states. The model can be used to simulate LV wall thickening (hypertrophy) or LV circumferential wall lengthening (dilated cardiomyopathy), as well as changes in effective contractility that create modified pressure–volume relationships. The derivation of the baseline model is provided in Section 2, and the baseline structural response is reviewed in Section 3. Our modified constitutive equation for microstructural change and the altered structural response are derived in Section 4. The results of a numerical parametric study and discussion of its implications are then provided in Section 5.

2. The baseline model

This section presents the essential ingredients of the simplified model of the LV developed in Nardinocchi et al. [10,11], with a slight modification. The LV is modeled as a thin-walled hollow sphere that undergoes a radially symmetric motion due to muscle tissue contraction and a uniform time-dependent pressure p applied to its internal surface from the contained blood. The hollow sphere is idealized as a membrane with slack, or unpressurized, radius R_s. In the current state, its radius is r, the pressure over its internal surface is p, and the biaxial Cauchy tensile stress is σ distributed uniformly over the wall thickness h. Except where needed for clarity, explicit indication of time as an independent variable is suppressed for notational simplicity. Force balance of a hemispherical portion of the hollow sphere in the current configuration is expressed by the relation

$$p = 2\frac{\sigma h}{r}. \tag{1}$$

Although the LV wall is a composite material formed of muscle, extracellular matrix (collagen), and vasculature, it is treated as an equivalent homogeneous, isotropic material that supports a mean equi-biaxial stress σ. Thus, this heart model is composed of material elements, each of which is envisioned as a representative volume element (RVE) whose response can, in concept, be derived from that of its components by a micromechanical analysis. Thus, from here on we use the terminology 'muscle tissue' or 'material element' interchangeably to represent the homogenized material element of muscle tissue.

Each material element undergoes equal biaxial circumferential stretch. Its reference configuration is considered to be stress free and is described as being slack. Let L_s and ℓ denote the slack and current lengths, respectively.[1] The circumferential stretch ratio

$$\lambda = \frac{\ell}{L_s} \tag{2}$$

is associated with the deformation from the slack state to the current state, called the visible (or total) deformation. The muscle fiber will contract when subjected to an electrochemical stimulus if it is not constrained by an externally applied force. Let L_c denote the (time-dependent) contracted length when the muscle tissue is stimulated but unstressed. The circumferential contraction stretch ratio

$$\lambda_c = \frac{L_c}{L_s} \tag{3}$$

is associated with the deformation induced by the stimulus, called the active deformation ($0 < \lambda_c \le 1$). The circumferential stretch ratio

$$\lambda_m = \frac{\ell}{L_c} \tag{4}$$

is associated with the deformation from the contracted state to the current state. It is called the mechanical stretch ratio, which is the passive, purely elastic stretching associated with the applied stress. Eqs. (2)–(4) imply the decomposition

$$\lambda = \lambda_m \lambda_c, \tag{5}$$

which interprets the total deformation as an active deformation from the slack to a stress free contracted state followed by a mechanical deformation from the contracted state to the current state.

The slack, contracted and visible lengths of the muscle tissue correspond to distinct radii and contained volumes of the spherical surface, which enter into the pressure–volume relationship. This correspondence leads, on using $v = 4\pi r^3/3$, to the following relations:

$$\lambda_c = \frac{R_c}{R_s} = \left(\frac{V_c}{V_s}\right)^{1/3}, \tag{6a}$$

$$\lambda_m = \frac{r}{R_c} = \left(\frac{v}{V_c}\right)^{1/3} = \frac{1}{\lambda_c}\left(\frac{v}{V_s}\right)^{1/3}, \tag{6b}$$

[1] Upper case variables are used to denote reference, or stress free, configurations, while corresponding lower case variables denote current (loaded) configurations.