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Response evolutionary power spectrum determination of chain-like MDOF non-linear structural systems via harmonic wavelets

Fan Kong [a,*,1], Pol D. Spanos [b], Jie Li [c], Ioannis A. Kougioumtzoglou [d]

[a] School of Civil Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China
[b] Department of Mechanical Engineering, Rice University, 6100 Main, Houston, Texas 77005-1827, United States
[c] State Key Laboratory of Disaster Reduction in Civil Engineering, School of Civil Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China
[d] Institute for Risk and Uncertainty, University of Liverpool, Liverpool L69 3GQ, United Kingdom

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ABSTRACT

A harmonic wavelets based technique is developed for determining the evolutionary power spectrum (EPS) matrix of the response of non-linear chain-like MDOF structural systems subject to a multi-component non-stationary stochastic excitation. Specifically, first a relationship between the evolutionary power spectrum matrices of the excitation and of the system response is derived by using a recently proposed locally stationary wavelet based representation of non-stationary stochastic processes. The relationship can be construed as a direct extension of the celebrated spectral input–output relationship of the linear stationary random vibration theory. Further, a harmonic wavelets based statistical linearization technique is proposed for the case of MDOF non-linear systems with chain-like architecture systems and hysteretic non-linearities. Numerical examples include MDOF non-linear systems comprising the versatile Bouc–Wen hysteretic model. Pertinent Monte Carlo simulations demonstrate the reliability of the technique.

1. Introduction

Infrastructure systems are often subjected to extreme loads such as earthquakes, strong winds, and ocean waves which are inherently stochastic and non-stationary. Further, under these non-stationary stochastic loads, structural systems often exhibit non-linear behavior. In this regard, quantifying the non-linear stochastic response behavior of such complex structures subject to non-stationary excitations is a sustained challenge in the field of random vibrations.

From a historical perspective, research in stochastic dynamics can be traced back to the study of the Brownian motion by Einstein in 1905 [1], and has since experienced a remarkable growth for over 100 years. Further, the tools of random vibrations theory, starting with Crandall in 1958 [2], have been applied to diverse engineering problems; see, for instance, Roberts and Spanos [3]. Nevertheless, despite the significant level of maturity that random vibration theory has reached for the case of linear systems, the case of non-linear systems remains a challenging one [4]. Indicatively, approaches for addressing non-linear problems in stochastic dynamics include statistical linearization/non-linearization techniques [5]; stochastic averaging [6]; Gaussian/non-Gaussian closure schemes [7–9]; maximum entropy principles [10]; path integral techniques [11–13], and several other numerical approximate analytical concept [14,15]. It is noted that certain of the techniques are based on the Markovian property/assumption of the response, whereas in most cases their applicability/performance is problem dependent. Regarding the statistical linearization/non-linearization technique, it has been shown that it can capture reliably the second order response statistics of non-linear MDOF systems; see Roberts and Spanos [3] and Socha [16] for a more detailed presentation. Nevertheless, it can be argued that it cannot be generalized to account for evolutionary excitation power spectra with time-varying frequency content in a straightforward manner. In this regard, note that a non-linear MDOF system dimension reduction approach has been proposed recently which utilizes the statistical linearization technique in a quasi-stationary manner to address cases of non-stationary excitations [14].

Note that potent signal processing techniques, such as wavelet analysis[17], which have been recently applied in diverse engineering problems (e.g., joint time–frequency analysis ([18,19]) can estimate efficiently the EPS of stochastic processes based on available realizations. In this regard, early researches on wavelet based representations of the excitation and of the system response include the work by Basu and Gupta [20,21], and Tratskas and

* Corresponding author. Tel.: +86 180 6268 8562.
E-mail addresses: f_kong@whut.edu.cn (F. Kong), spanos@rice.edu (P.D. Spanos), lijie@tongji.edu.cn (J. Li), kougioum@liverpool.ac.uk (I.A. Kougioumtzoglou).
[1] Present address: School of Civil Engineering and Architecture, Wuhan University of Technology, 122 Luoshi Road, Wuhan 430070, China

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Spanos [22]. Further researches are based on the combination of statistical linearization and wavelet analysis to deal with non-linearities. Specifically, Basu and Gupta [23–25] proposed an equivalent linear stiffness expression based on the wavelet transform of the non-linear function. However, it can be argued that this treatment negates, in a sense, the joint time-frequency capabilities of the wavelet transform since the equivalent linear element is only time-dependent (as opposed to both time and frequency dependent); see also Spanos and Kougioumtzoglou [26]. Further, it is noted that the above treatment refers to realizations of stochastic processes; that is, there is no stochastic framework for representing stochastic processes via wavelets.

Recently, Spanos and Kougioumtzoglou [26] developed a harmonic wavelets based statistical linearization technique for obtaining the response evolutionary power spectrum (EPS) of non-linear single-degree-of-freedom (SDOF) systems. The proposed technique is based on a mathematical representation of non-stationary stochastic processes via the locally stationary wavelet (LSW) process representation proposed by Nason et al. [27]. Further, it is noted that in many practical problems the system of concern comprises of a number of lumped masses, or nodal points, interconnected by non-linear elements whose behavior depends only upon the relative coordinates between adjacent points. In these situations, where one has a chain-like structural system, it is convenient to set as the coordinate vector in the non-linear equation of motion the vector of the relative displacements between the nodal points. In this paper, therefore, the aforementioned technique [26] is extended/generalized to be applicable for the chain-like MDOF systems with even complex hysteretic behavior. Numerical examples pertain to the versatile Bouc–Wen model, and comparisons with proper Monte Carlo simulations demonstrate the reliability of the technique.

2. Harmonic wavelets elements and non-stationary stochastic process representation

2.1. Harmonic wavelets

The family of harmonic wavelets (HW), first emphasized by Newland [28], has been widely used for engineering dynamics applications. For example, Spanos et al. [29] employed HWs for estimating the EPS of non-stationary stochastic processes. Spanos and Tratskas [22] suggested a HW based scheme for determining the response EPS of MDOF system. Further, the time–frequency localization features of the HW have been utilized for damage detection applications as well [19]; see also Failla and Spanos [30] for a broad perspective.

GHWs possess a band-limited, box-like frequency spectrum. A wavelet of scale $(m,n)$and time-shift k has a representation in the frequency domain as

$$\Psi^{G}_{(m,n),k}(\omega)=\begin{cases}\frac{1}{(n-m)\Delta\omega}e^{-\mathrm{i}\frac{\omega kT_0}{n-m}}, & m\Delta\omega\le\omega<n\Delta\omega\\ 0, & \text{otherwise}\end{cases},\tag{1}$$

where the scale (m, n) and time-shift k parameters are considered to be positive numbers, with $\Delta\omega=2\pi/T_0$, where T_0 is the time duration of the signal under consideration. The inverse Fourier transform of Eq. (1) gives the HW representation in the time domain as

$$\psi^{G}_{(m,n),k}(t)=\frac{\exp\left[\mathrm{i}n\Delta\omega\left(t-\frac{kT_0}{n-m}\right)\right]-\exp\left[\mathrm{i}m\Delta\omega\left(t-\frac{kT_0}{n-m}\right)\right]}{\mathrm{i}(n-m)\Delta\omega\left(t-\frac{kT_0}{n-m}\right)}.\tag{2}$$

Note that Spanos et al. [29] have also suggested an alternative equivalent time-domain representation involving the phase and magnitude of the HW; see also Newland [28] for more details on the wavelet transform and the corresponding inverse transform.

2.2. Locally stationary wavelet (LSW) representation of stochastic processes

The most widely used model of stochastic processes is the celebrated Wold-Cramer representation associated with the Priestley's EPS definition [31,32]. The recently developed representation of stochastic processes by the locally stationary wavelet (LSW) as proposed by Nason et al. [27], is adopted herein. Note that in comparison with the Wold–Cramer model where non-stationarity is somewhat heuristically imposed by defining a “slowly” time-varying modulating amplitude function, non-stationarity in the LSW model is featured in a somewhat more natural manner by exploiting the localized wavelet amplitude. According to [26], if the GHW is chosen for the LSW stochastic process representation, then a n_d-dimensional vector-valued random process can be modeled as the summation of sub-processes defined at different scales and translation levels in the form

$$\mathbf{x}(t)=\sum_{(m,n)}\sum_{k}\mathbf{x}_{(m,n),k}(t),\tag{3}$$

where $\mathbf{x}_{(m,n),k}(t)$ can be defined as the combination of localized monochromatic functions weighted by random vectors. That is,

$$\mathbf{x}_{(m,n),k}(t)=\mathbf{a}_{(m,n),k}\cos\left[\omega_{c,(m,n),k}\left(t-\frac{kT_0}{n-m}\right)\right]+\mathbf{b}_{(m,n),k}\sin\left[\omega_{c,(m,n),k}\left(t-\frac{kT_0}{n-m}\right)\right].\tag{4}$$

Note that Eq. (4) is localized in the sense that its effective support is limited to the intervals $\omega\in[m\Delta\omega,\ n\Delta\omega]$ and $t\in[kT_0/(n-m),\ (k+1)T_0/(n-m)]$, with $\omega_{c,(m,n),k}$ being the central frequency of a given frequency band. That is,

$$\omega_{c,(m,n),k}=\frac{(n+m)}{2}\Delta\omega.\tag{5}$$

Further, the symbols $\mathbf{a}_{(m,n),k}$, $\mathbf{b}_{(m,n),k}$ represent statistically independent random vectors with zero-mean and variance equal to

$$\mathrm{E}(\mathbf{a}_{(m,n),k}\mathbf{a}^{\mathrm{T}}_{(m,n),k})=\mathrm{E}(\mathbf{b}_{(m,n),k}\mathbf{b}^{\mathrm{T}}_{(m,n),k})=2\mathbf{S}^{\mathbf{xx}}_{(m,n),k}(n-m)\Delta\omega,\tag{6}$$

where the superscript $^{\mathrm{T}}$ denotes transposition of a matrix. The local auto/cross EPS of the process $\mathbf{x}_{(m,n),k}(t)$.

$$\mathbf{S}^{\mathbf{xx}}_{(m,n),k}=\begin{bmatrix}S^{x_1,x_1} & S^{x_1,x_2} & \cdots & S^{x_1,x_{n_d}}\\ S^{x_2,x_1} & S^{x_2,x_2} & \cdots & S^{x_2,x_{n_d}}\\ \vdots & \vdots & \ddots & \vdots\\ S^{x_{n_d},x_1} & S^{x_{n_d},x_2} & \cdots & S^{x_{n_d},x_{n_d}}\end{bmatrix}_{(m,n),k}\tag{7}$$

is a Hermitian EPS matrix, and the entries $S^{x_i,x_j}_{(m,n),k}$are the two-sided localized auto/cross spectra of the processes $x_i(t)$ and $x_j(t)$ at different scales and translation levels $i,j=1,2,\cdots,n_d$. Next, utilizing the orthogonality conditions of the monochromatic functions

$$\int_{\frac{kT_0}{n-m}}^{\frac{(k+1)T_0}{n-m}}\cos\left[\omega_{c,(m,n),k}\left(t-\frac{kT_0}{n-m}\right)\right]\cos\left[\omega_{c,(p,q),l}\left(t-\frac{lT_0}{n-m}\right)\right]\mathrm{d}t=\begin{cases}T_0/2(n-m), & \text{for }(m,n)=(p,q),\ k=l\\ 0, & \text{otherwise}\end{cases},\tag{8}$$

$$\int_{\frac{kT_0}{n-m}}^{\frac{(k+1)T_0}{n-m}}\sin\left[\omega_{c,(m,n),k}\left(t-\frac{kT_0}{n-m}\right)\right]\sin\left[\omega_{c,(p,q),l}\left(t-\frac{lT_0}{n-m}\right)\right]\mathrm{d}t=\begin{cases}T_0/2(n-m), & \text{for }(m,n)=(p,q),\quad k=l\\ 0, & \text{otherwise}\end{cases}\tag{9}$$