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Non-linear vibrations and stability of a periodically supported rectangular plate in axial flow

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ABSTRACT

In the present study, the geometrically non-linear vibrations of thin infinitely long rectangular plates subjected to axial flow and concentrated harmonic excitation are investigated for different flow velocities. The plate is assumed to be periodically simply supported with immovable edges and the flow channel is bounded by a rigid wall. The equations of motion are obtained based on the von Karman non-linear plate theory retaining in-plane inertia and geometric imperfections by employing Lagrangian approach. The fluid is modeled by potential flow and the flow perturbation potential is derived by applying the Galerkin technique. A code based on the pseudo-arc-length continuation and collocation scheme is used for bifurcation analysis. Results are shown through bifurcation diagrams of the static solutions, frequency-response curves, time histories, and phase-plane diagrams. The effect of system parameters, such as flow velocity and geometric imperfections, on the stability of the plate and its geometrically non-linear vibration response to harmonic excitation are fully discussed and the convergence of the solutions is verified.

1. Introduction

Flow-induced vibrations of plates are a major problem in many engineering applications including aerospace, aeronautics, automotive, nuclear and naval industries. In these challenging applications, in order to accurately predict the non-linear response of the plate, it is of significant importance to consider numerical models that take into account (i) non-linear effects such as large structural deflections, and (ii) fluid–structure interactions.

The state of art research on geometrically non-linear vibrations of plates can be found in the book by Amabili [1]. In a series of papers, Amabili [2–5] profoundly investigated non-linear vibrations of thin isotropic rectangular plates. In particular, large amplitude vibrations of plates with different boundary conditions based on a Lagrangian approach were studied in Ref. [2] by using an arc-length continuation technique. Theory and experiments for thin plates with non-conventional boundary conditions and geometric imperfections were presented in Ref. [3]. The effect of temperature variations on non-linear vibrations of plates with clamped edges was investigated in Ref. [4], and the effect of concentrated masses on the large amplitude response of rectangular plates was presented in Ref. [5]. Moreover, the experimental results for non-linear vibrations of fully clamped rectangular plates with concentrated masses were presented by Amabili and Carra [6], and the theoretical and experimental results for large amplitude vibrations of completely free imperfect rectangular plates were given in Ref. [7]. Finite Element methods (FEM) have also been extensively used for studying large-amplitude vibrations of rectangular plates. For instance, Ribeiro and Petyt [8–10] used the hierarchical finite element method (HFEM) to investigate large amplitude vibrations of fully clamped plates with [8] and without internal resonance [9,10]. Ribeiro [11] used the same approach to study the non-linear forced response of simply supported plates with immovable edges. Touzé et al. [12] studied the transition to chaos in non-linear vibration of rectangular plates.

The literature related to linear vibrations of plates coupled to fluid is quite extensive (see e.g. [13–17]). In particular, the majority of the approximate analytical methods that are used to study flow-induced vibrations are based on the assumption attributed to Ref. [13] that the vibration modes of the structure in contact with still fluid (wet modes) are the same as those in vacuo (dry modes). In fact, it is based on this assumption that the so-called non-dimensional added virtual mass incremental factors (NAVMI) can be used to estimate the natural frequencies of the plate in still fluid from the natural frequencies in vacuo as shown by Kwak and Kim [14] and Kwak [15]. The linear hydroelastic theory including dry and wet analyses has also been used by Fu and Price [16] to study vibrations of a cantilevered plate partially and completely immersed in fluid. In the dry analysis, the dynamic characteristics

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of the cantilevered plate were determined in the absence of internal damping and external forces. However, in the wet analysis, a boundary value problem was formulated and the fluid actions were treated as generalized external forces. Amabili and Kwak [17] removed the simplified assumption of identical wet and dry modes and obtained the mode shapes of the coupled system via Rayleigh–Ritz approach. Experiments on large amplitude vibrations of a bottom circular plate of a water-filled container were presented by Chiba [18].

In case of flowing fluid, in addition to the inertia effect of the fluid, the stiffness of the coupled plate-flow system decreases with the flow speed, resulting eventually to instability. Moreover, the presence of gyroscopic terms in the equations of motion gives rise to complex modes and therefore different points of the plate do not oscillate in-phase anymore. Guo and Païdoussis [19] used Galerkin approach to study the hyrdoelastic instabilities of parallel assemblies of rectangular plates coupled to flow. They found that divergence and coupled mode flutter may occur for plates with any type of end supports, while single-mode flutter only arises for non-symmetrically supported plates. Kerboua et al. [20] used a different approach based on the combination of FEM and Sander′s shell theory to determine the natural frequencies of rectangular plates in contact with flowing fluid. In their study the velocity potential and Bernoulli′s equation were used to express the fluid pressure acting on the structure. Tubaldi and Amabili [21] derived the eigenfrequencies and complex modes of an infinite plate periodically supported and coupled to flowing fluid using the Rayleigh–Ritz method. They found that for sufficiently high flow velocities the system becomes statically unstable. Implicit in the authors′ analysis was the assumption that the plate deflection was the same between any two successive supports in the flow direction, aside from a phase change (change in sign) between two successive supports and the next set of supports, i.e. from one "bay" to the next. Indeed, for the low speed case treated by the authors, the instability is divergence at low speeds (rather than flutter which occurs at supersonic speeds) and the divergence instability is dominated by a single structural mode. A general aerodynamic case of a single elastic plate embedded in a rigid surface (baffle) has been treated in Dowell [22]. Dowell also discussed the case of both finite and infinite plates on periodic supports for high supersonic flow [23]. As the number of bays becomes larger, he found that the flow velocity at which flutter occurs decreases. In supersonic flow the elastic plate deflections increase from one bay to the next bay and this must be taken into account. For a finite square panel, Dowell [24] found that at high Mach number the flutter frequency is between the first and second panel natural modal frequencies while over the subsonic range of Mach number the flutter frequency rapidly falls to zero and the panel diverges rather than flutters.

The literature related to non-linear studies of plates coupled to flowing fluid is scarce. Non-linear flutter of rectangular plates was investigated by Dowell [25,26]. Ellen [27] studied the asymptotic non-linear stability of simply supported rectangular plates subjected to incompressible flow (on one side only) considering both structural and fluid-dynamic non-linearities. The analysis was based on single-mode Galerkin approach and it was shown that fluid-flow non-linearities introduce a subcritical instability while the stabilizing structural non-linearities have a dominant effect in controlling the overall non-linear behavior. Lucey et al. [28] examined the dynamics of a finite length plate, mainly in post-divergence regime where coupled-mode flutter may arise. The flow was considered to be inviscid and the solution of the coupled problem was obtained by boundary-element and finite-difference method. The unsteady interaction between a simple elastic plate and a mean flow has a number of interesting features such as the existence of negative-energy waves (NEWs). Indeed by introducing the concept of modal wave energy, Landahl [29] and Benjamin [30,31] showed that over a range of frequencies neutral modes with negative wave energy exist (also named class A waves by Benjamin). In particular, Landahl [29] explained the seemingly paradoxical result that damping destabilizes class A waves by studying the flutter of an infinite panel in incompressible potential flow. It was shown that these waves are associated with a decrease of the total kinetic and elastic energy of the fluid and the wall, so that any dissipation of energy in the wall will only increase the wave. It was also found that the Kelvin–Helmholtz type of instability will occur when the effective stiffness of the panel is too low to withstand the pressure forces induced on the wall. Using the same concept of modal wave energy, Peake [32] studied the non-linear stability of plates for heavy fluid loading considering both plate and fluid non-linearities analytically. Also in this case it was found that the instability may arise if the destabilizing force due to the fluid loading exceeds the restoring stiffness of the plate.

Yao and Li [33] studied the non-linear dynamic characteristics of a simply supported laminated composite plate with geometric non-linearity in incompressible subsonic flow using the von Karman theory. The corresponding bifurcation diagrams of the laminated plate showed pitchfork bifurcations for different ply angles once the flow velocity was increased.

The present study aims to extend the recent work of Tubaldi and Amabili [21] by studying non-linear vibrations and stability of thin rectangular plates with immovable edges coupled to axial flow. The plate is periodically supported in both directions so that it is composed of an infinite number of supported rectangular plates with slope continuity at the edges and is immersed in axial flow on its upper side. In the case of flat plates, bifurcation diagrams with respect to the flow velocity are depicted and show a static loss of stability due to a pitchfork bifurcation. When geometric imperfections are taken into account, the pitchfork bifurcation disappears and the system presents a continuous post-buckling configuration. The non-linear dynamic behavior of rectangular plates in axial flow under external harmonic excitation is also investigated. The non-linear vibration response at different flow velocities is studied by using a code based on pseudo-arclength continuation and collocation scheme. Convergence of the solution with the number of generalized coordinates is numerically verified. A hardening type non-linearity has been found for the entire flow velocity range explored in the case of flat plate. Conversely, an initial softening behavior turning to strong hardening for large vibration amplitudes have been obtained for imperfect plates. Moreover, modal interactions in the response of the fundamental mode with higher modes are detected for certain frequency ranges.

2. Problem definition

The system under investigation, shown in Fig. 1, consists of an infinitely wide and infinitely long thin plate made of isotropic homogeneous material subjected to an inviscid axial flow on its upper surface. The plate is taken in the proximity of a rigid wall as shown in the Fig. 1(a). A right-handed rectangular Cartesian reference system $(O; x, y, z)$ is considered with the x, y plane coinciding with the middle surface of the plate in its initial undeformed configuration and the z-axis normal to it. The distance between the plate and the rigid wall is denoted by H and U is the undisturbed flow velocity of the axial flow. A geometric imperfection w_0 associated with zero initial stress is taken into account. The plate is assumed to be simply supported with immovable edges and therefore the following boundary conditions