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Non-linear free vibration analysis of laminated cylindrical shells under static axial loading including accurate satisfaction of boundary conditions

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ABSTRACT

The effect of a static preload on the non-linear vibration behaviour of laminated anisotropic shells is studied. Starting point of the analysis is a perturbation approach developed earlier to analyse the non-linear vibration behaviour of imperfect general structures under static preloading. This method is based on a perturbation expansion for both the frequency parameter and the dependent variables, and can be used to study the effects of geometric imperfections, a static fundamental state, and a non-trivial static state on the linearised and non-linear vibrations of cylindrical shells. The theory has been incorporated within a semi-analytical framework, based on Donnell-type governing equations.

The approach is applied in the non-linear vibration analysis of laminated, anisotropic cylindrical shells. Special characteristics of the non-linear free vibration behaviour of statically, axially loaded cylindrical shells are shown for a specific composite shell. Internal resonances corresponding to interactions between first order modes and second order modes are found in the analysis. Novel aspect of the current work is that these phenomena are considered within a framework in which effects of various types of boundary conditions, including the effect of non-linear static state deformation due to the application of the static load, are accurately taken into account.

1. Introduction

The non-linear vibration behaviour of cylindrical shells is an important area in the dynamic analysis of structures [1,2]. Obtaining information about the free vibration non-linear amplitude-frequency relation can be regarded as a preliminary step before carrying out a steady-state non-linear response analysis or a non-linear transient analysis.

Early work on the non-linear vibrations of composite cylindrical shells was done by Iu and Chia (using a semi-analytical approach) [12] and by Ganapathi and Varadan (using the Finite Element method) [11]. More recently, Amabili compared different shell theories for the analysis of the non-linear vibrations of laminated circular cylindrical shells [3] and studied internal resonances in geometrically non-linear forced vibrations of laminated circular cylindrical shells [4] using a recently developed higher-order shear deformation theory. The present author studied the effect of static loading and imperfections on the non-linear vibrations of laminated cylindrical shells using a limited number of assumed modes in [15] and the effect of shell length and layer orientation on the single mode non-linear vibrations of laminated shells using an approach in which the boundary conditions are accurately satisfied [14].

The effect of static axial loading on the non-linear vibration of shells has since long been identified as an important issue [10]. Investigations including the effect of static in-plane (axial) loading have been carried out in studies on the behaviour of shells under parametric excitation, e.g. by Pellicano [16]. Using the “Simplified Analysis” introduced in [15], main trends of the non-linear vibration behaviour of shells under static loading can be captured. However, the effects of static axial loading have not yet been systematically studied within a framework in which various types of boundary conditions, including the effects of non-linear static state deformation due to the application of the static load, are accurately taken into account. This is the novel aspect of the current work.

Objective of the present work is to illustrate several key aspects related to the behaviour of non-linear vibrations of statically loaded shells. In the analysis, the specified boundary conditions at the shell edges can be satisfied rigorously, and both the static and dynamic non-linear stress and deformation state can be accurately determined through the numerical integration of

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Nomenclature

$A^*_{ij}, B^*_{ij}, D^*_{ij}$ semi-inverted stiffness matrices
c $= \sqrt{3(1-\nu^2)}$
E reference Young's modulus
f_0 fundamental state stress function component
f_1, f_2 first-order stress function components
$f_\alpha, f_\beta, f_\gamma$ second-order stress function components
F Airy stress function
$F^{(0)}, F^{(1)}, F^{(2)}$ stress functions for 0th-, 1st-, and 2nd-order state
h reference shell wall thickness
L shell length
$L_{A^*}, L_{B^*}, L_{D^*}$ linear operators
L_{NL} non-linear operator
n number of full waves in circumferential direction
N_0 applied axial load ($N_0 = -N_x \ (x = L)$)
N_{cl} classical buckling load ($N_{cl} = (Eh^2)/(cR)$)
R radius of shell
T_0 applied torque ($T_0 = N_{xy} \ (x = L)$)
u, v, W axial, circumferential and radial displacement
w normalised radial displacement ($w = W/h$)
w_0 fundamental state radial displacement component
w_1, w_2 first-order radial displacement components
$w_\alpha, w_\beta, w_\gamma$ second-order radial displacement components
W_ν, W_p, W_t generalised Poisson's expansions
$W^{(0)}, W^{(1)}, W^{(2)}$ radial displacements for 0th-, 1st-, and 2nd-order state
$\overline{W}$ initial radial imperfection
x, y, z axial, circumferential and radial coordinate, respectively
θ non-dimensional circumferential coordinate ($\theta = y/R$)
θ_k orientation of kth layer
$\kappa_x, \kappa_y, \kappa_{xy}$ curvature changes and twist, respectively
λ normalised axial load ($\lambda = (cR)/(Eh^2)N_0$)
ν reference Poisson's ratio
$\overline{\rho}$ averaged specific mass
$\overline{\omega}$ normalised radial frequency ($\overline{\omega} = R\sqrt{(\overline{\rho}h/A_{22})}\ \omega$)
$\overline{\omega}_{c0}$ normalised linear natural frequency of the unloaded perfect shell

Indices

$()_0$ static fundamental state
$\tilde{()}$ static state
$\hat{()}$ dynamic state
$()_,$ partial differentiation w.r.t. variable following the comma

a two-point boundary value problem. The characteristics of the non-linear free vibration behaviour of loaded shells will be shown for a specific reference shell. Important effects of the applied static load and of the non-linearity of the axisymmetric static fundamental state on the non-linear free vibrations of a laminated cylindrical shell, such as modal interactions, will be highlighted.

2. Governing equations

The method used in the present work to study the effects of static loading on the free vibration behaviour of cylindrical shells, denoted in the following as "Extended Analysis" [14], is briefly recapitulated for completeness. The shell geometry and the applied loading are defined in Fig. 1. The shell geometry is characterised by its length L, radius R and thickness h.

Assuming that the radial displacement W is positive inward (see Fig. 1) and introducing an Airy stress function F as $N_x = F_{,yy}$, $N_y = F_{,xx}$ and $N_{xy} = -F_{,xy}$, where N_x, N_y and N_{xy} are the usual stress resultants, then the Donnell-type non-linear imperfect shell equations for a general anisotropic material can be written as

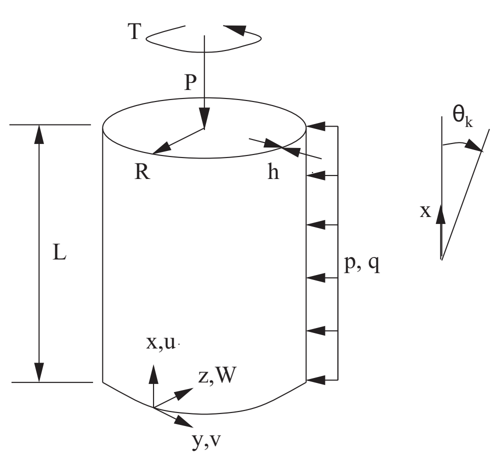

Fig. 1. Shell geometry, coordinate system and applied loading.

$$L_{A^*}(F) - L_{B^*}(W) = -\frac{1}{R}W_{,xx} - \frac{1}{2}L_{NL}(W, W + 2\overline{W}) \quad (1)$$

$$L_{B^*}(F) + L_{D^*}(W) = \frac{1}{R}F_{,xx} + L_{NL}(F, W + \overline{W}) + p - \overline{\rho}hW_{,tt} \quad (2)$$

where the variables W and F depend on the time t, R is the shell radius, $\overline{W}$ is an initial radial imperfection, $\overline{\rho}$ is the (averaged) specific mass of the laminate, h is the (reference) shell thickness, p is the (effective) radial pressure (positive inward), and $\overline{\rho}hW_{,tt}$ is the radial inertia term. Donnell-type equations are assumed to be accurate for vibration modes containing a sufficiently high number of circumferential waves. It is noted that in-plane inertia of the (predominantly) radial modes is neglected. This is also expected to be an appropriate assumption for vibration modes which correspond to a sufficiently high number of circumferential waves. It is noted further, that in the current Donnell-type shell formulation the effect of transverse shear deformation as well as the corresponding rotatory inertia effect are neglected. For sufficiently thin shells and modes for which the wave length is not too small, this approximation is assumed to be reasonable.

The fourth-order linear differential operators L_{A^*}, L_{B^*}, and L_{D^*} depend on the stiffness properties of the laminate and are defined in [14], and the non-linear operator defined by

$$L_{NL}(S, T) = S_{,xx}T_{,yy} - 2S_{,xy}T_{,xy} + S_{,yy}T_{,xx} \quad (3)$$

reflects the geometric non-linearity.

The shell can be loaded by axial compression P, radial pressure p and counter-clockwise torsion T (Fig. 1), both statically ($\tilde{P}$, $\tilde{p}$, $\tilde{T}$) and dynamically ($\hat{P}$, $\hat{p}$, $\hat{T}$). The equations governing the non-linear dynamic behaviour of a cylindrical shell vibrating about a non-linear static state will be derived, by expressing both the displacement W and the stress function F as a superposition of two states,

$$W = \tilde{W} + \hat{W} \quad (4)$$