International Journal of Non-Linear Mechanics 66 (2014) 87–95

Contents lists available at ScienceDirect

International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm

Creep and damage in shells of revolution under cyclic loading and heating

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ARTICLE INFO

Available online 20 February 2014

Keywords:
Creep
Damage
Shell of revolution
Cyclic loading and heating
Asymptotic expansions
FEM code

ABSTRACT

Creep of cyclically loaded thin shells of revolution and their fracture due to creep and fatigue mechanisms are studied. Creep–damage equations for steels and nickel-based alloys are built by the use of scalar damage parameter. Constitutive equations were derived using the method of asymptotic expansions and averaging over a period of cyclic loading. The cases of fast and slow varying of temperature and loading are regarded. General problem statement and method for solution of creep problems at cyclic loading are presented. Strain–stress state in shell structures is determined by the use of homemade FEM creep–damage code, where the finite element of conical shell is used. Results of creep–damage problem for conical panel are discussed.

1. Introduction

Cyclic loading and heating are often met in industrial applications. As a rule, at elevated temperatures they essentially aggravate the working conditions of structural elements because of increasing of creep strain rate as well as decreasing of time to fracture values. Because of the long duration and high costs of experimental investigations of structural elements; their numerical simulations are necessary. Despite the fact that solutions of linear problems can be obtained numerically by the use of FEM software, modeling of non-linear problems meet considerable difficulties. One of them is adequate formulation of constitutive equations, especially for complex cases of deformation processes like cyclic loading. The case of joint action of static and cyclically varying loading have long attracted the attention of scientists and engineers. The more general case of stresses which exceed the yield limit has been studied in detail [1–6]. The interest to such phenomenon continues unabated and interaction between low-cycle fatigue and creep effects for the cases of cyclic varying of loads and temperatures attracts the attention of researchers over the last decade [7–10].

However, operational conditions of many structural elements which work in aviation, power and chemical industry are designed so that arising stresses do not exceed yield limit. This fact allows the use of simplified approaches, one of the most effective is averaging of resulting curves of strain growth and damage accumulation. If the number of cycles is high, the changes of these parameters through a cycle can be regarded as negligible, but the cyclic varying of stresses and temperatures influences on the rates of strain and damage parameter. For the case of cyclic variation of temperatures or stresses the approach connected with averaging of resulting creep strain curves has been shown to be effective [11–15]. Such clear physical procedure got its mathematical justification by the way of using the methods of asymptotical expansions and averaging over a period of stress or temperature variation [15–17].

Present investigations which have been done in this direction were focused on particular cases of cyclic creep–damage problems. The phenomenon of so-called dynamic creep [12], when cyclic variation of stresses due to forced vibrations is imposed on the static stress component has been investigated for plane creep problems [16]. Further interaction between damage accumulation from dynamic creep process and high cycle high temperature fatigue has been analyzed for thin shells of revolution in [17]. The situations of slow variation of stresses and temperatures were studied in [18,19].

However, a certain number of structural elements is loaded and heated by complex programs. In addition their forced vibrations cannot be completely suppressed. That is why the formulation of the generalized approach to description of creep–damage processes allowing consideration of complex programs of loading and heating can be regarded as useful for engineering applications.

The main goal of this paper is to discuss the derivation method for creep problems with complex cyclic loading or heating programs. The examined problem can be considered as an

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http://dx.doi.org/10.1016/j.ijnonlinmec.2014.02.005
0020-7462

example of the application of this technique. With this example in mind the other practical problems with different types of cyclic programs can be solved. However, the most common cases are considered.

The paper contains the generalized constitutive model in which the list of possible effects related to cyclic character of loading and heating is considered. The method of solution is presented first for general 3D case and further for thin shells of revolution. FEM approach with time numerical integration was used.

2. Constitutive model for different cases of cyclic loading

The mostly common case which includes the cycles of loading/unloading and heating/cooling is regarded. Let us first regard at the behavior of the material at uniaxial stress state. Results will be further generalized to the complex stress state.

It is well known that mechanical behavior of the specimen subjected by cyclic loading significantly depends upon its frequency. The cyclic creep–damage processes in a solid which are originated by the action of temperature field can be divided into the action of low or high cycle loading or heating.

Equations for rates of creep strain c and damage parameter ω for the uniaxial loading that are used in the Bailey–Norton and Rabotnov–Kachanov [20] forms:

$$\dot{c} = B\frac{(\sigma)^n}{(1-\omega)^k}, \quad \dot{\omega} = D\frac{(\sigma)^r}{(1-\omega)^l} \tag{1}$$

where B, n, k, D, r, and l are the material constants.

Let us limit by this constitutive model.

For all cases of cyclic loading total stress in the specimen is presented as a sum of constant part σ and varying part σ^1 : $\overline{\sigma} = \sigma + \sigma^1$. Just the same the temperature has constant part T and varying part T^1 : $\overline{T} = T + T^1$.

Firstly let us regard the quasi static creep processes with long periods more than 1 s. Generally they can be presented as polyharmonic with the global period T_σ:

$$\overline{\sigma} = \sigma + \sigma^1 = \sigma\left(1 + \sum_{k=1}^{\infty} M_k \sin\left(\frac{2\pi k}{T_\sigma}t + \gamma_k\right)\right) \tag{2}$$

At first we consider that temperature has the constant value T.

We expand creep strain and the damage parameter into asymptotic series of the small parameter $\mu = T_\sigma / t_* \ll 1$ [21] and keep two terms of series:

$$c \cong c^0(t) + \mu c^1(\xi), \quad \omega \cong \omega^0(t) + \mu\omega^1(\xi) \tag{3}$$

where $c^0(t)$, $\omega^0(t)$, $c^1(\xi)$, $\omega^1(\xi)$ are the functions which coincide with basic creep–damage process in slow (0) and fast (1) time scales.

Used truncated asymptotic series have direct physical meaning: the first term corresponds to slow motion whereas the second describes the motion through a cycle.

Two times are considered: slow time t which varies from 0 to the time to rupture moment t_*; fast time $\tau = t/\mu$ or $\xi = \tau / T_\sigma$ $(0 \le \xi \le 1)$ [18]. After the substitution of Eq. (3) into Eq. (1) and further averaging [21,22] over the period of stress variation we obtain the expressions of creep strain and the damage parameter on the time interval $(0 \le \xi \le 1)$:

$$\langle c^0(\xi)\rangle = \int_0^1 c^0(t)d\xi \cong c^0(t), \quad \langle c^1(\xi)\rangle = \int_0^1 c^1(\xi)d\xi \cong 0, \tag{4}$$

$$\langle \omega^0(\xi)\rangle = \int_0^1 \omega^0(t)d\xi \cong \omega^0(t), \quad \langle \omega^1(\xi)\rangle = \int_0^1 \omega^1(\xi)d\xi \cong 0, \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (1) we obtain [18]:

$$\dot{c} = Bg_n(M_k)\frac{(\sigma)^n}{(1-\omega)^m}, \quad \dot{\omega} = Dg_r(M_k)\frac{(\sigma)^r}{(1-\omega)^l}, \quad \omega(0) = \omega_0, \omega(t_*) = \omega_*$$

$$g_n(M_k) = \int_0^1 \left(1 + \sum_{k=1}^{\infty} M_k \sin(2\pi k\xi)\right)^n d\xi,$$

$$M_k = \frac{\sigma^{ak}}{\sigma}, \quad g_r(M_k) = \int_0^1 \left(1 + \sum_{k=1}^{\infty} M_k \sin(2\pi k\xi)\right)^r d\xi. \tag{6}$$

The main result here is following: we escaped from the necessity of the integration over the cycle and obtained averaged cyclic creep–damage laws. Here the functions $g_n(M_k)$ and $g_r(M_k)$ reflect the influence of cyclic character of loading.

If we consider more general low cycle process when not only stress varies due to law (2) but the temperature also:

$$\overline{T} = T + T^1 = T\left(1 + \sum_{i=1}^{\infty} M_i^T \sin\left(\frac{2\pi i}{T_T}t + \gamma_i^T\right)\right), \quad M_i^T = T_i^a / T, \tag{7}$$

the creep–damage laws can be written in the following forms [20]:

$$\dot{c} = B\frac{\overline{\sigma}^n}{(1-\omega)^m}\exp\left(-\frac{U_c}{R\overline{T}}\right) = B(\overline{T})\frac{\overline{\sigma}^n}{(1-\omega)^m}, \tag{8}$$

$$\dot{\omega} = D\frac{\overline{\sigma}^r}{(1-\omega)^l}\exp\left(-\frac{U_d}{R\overline{T}}\right) = D(\overline{T})\frac{\overline{\sigma}^r}{(1-\omega)^l} \tag{9}$$

Here U_c and U_d are the constants which are equal to the values of activation energy in creep and creep damage processes, R is the universal gas constant. In the remainder we use the notations $Q = U_c/R$ and $\overline{Q} = U_d/R$.

Using the described asymptotic expansions method and the procedure of averaging over a periods of loading and heating for Eqs. (8) and (9), considering the results of similar procedures which led to Eq. (6), the low cycle creep–damage equations can be written in the following form:

$$\dot{c} = g_T(T)g_n(M_k)\frac{\sigma_i^n}{(1-\omega)^m}, \quad c(0) = 0, \tag{10}$$

$$\dot{\omega} = g_r(M_k)g_T^{\omega}(T)\frac{\sigma^r}{(1-\omega)^l}, \quad \omega(0) = \omega_0, \quad \omega(t_*) = \omega_* \tag{11}$$

where

$$g_T(T) = B\int_0^1 \exp\left(-\frac{Q}{T}\left(1 + \sum_{i=1}^{\infty} M_i^T \sin(2\pi i\xi)\right)^{-1}\right)d\xi, \quad M_i^T = \frac{T_i^a}{T},$$

$$g_T^{\omega}(T) = D\int_0^1 \exp\left(\frac{-\overline{Q}}{T}\left(1 + \sum_{i=1}^{\infty} M_i^T \sin(2\pi i\xi)\right)^{-1}\right)d\xi.$$

Now let us consider the case of mono-harmonic sine loading, with frequencies more than 1 Hz

$$\overline{\sigma} = \sigma + \sigma^a \sin \varpi t, \tag{12}$$

where σ^a and σ are the amplitude and the mean stress, respectively.

In this case when the frequencies of oscillations correspond to phenomenon of forced vibrations the processes of dynamic creep as well as the creep–high cycle fatigue interaction can occur [12]. The character of a running processes is different depending of the value of stress cycle asymmetry coefficient $A = \sigma^a/\sigma$. The processes of dynamic creep and creep–fatigue interaction are divided by the value of so-called critical stress cycle asymmetry coefficient A_{cr}. It is known from experiments [12] that the dynamic creep occurs at low values of A, which causes the substantial acceleration of creep strain and damage rates. By the use of the procedures of asymptotic expansions and averaging over a period of forced vibrations like (3) and (4), we obtain dynamic creep averaged