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# Reprint of: Effective bond length of FRP stiffeners ☆

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ABSTRACT

The problem of an elastic bar bonded to an elastic half space and pulled at one end is considered to model the performance of FRP strips glued to concrete or masonry substrates. If the bond is perfect, stress singularities at both bar-extremities do appear. These can be removed by assuming cohesive contact forces *à là* Baranblatt that annihilate the stress intensity factor. We show that the presence of such cohesive zones is crucial to predict the experimentally measured *Effective Bond Length* (EBL), i.e., the bond length beyond which no apparent increase of strength is attained. In particular, it is the cohesive zone at the loaded end of the stiffener, rather than that at the free end, that governs the phenomenon because the EBL coincides with the maximal length of such a zone. The proposed approach provides better estimates than formulas proposed in technical standards.



## 1. Introduction

A promising technique to strengthen concrete or masonry structures consists in gluing to them strips made of Fiber Reinforced Polymer (FRP). Experiments have provided a wealth of evidence that the most frequent failure mode for this arrangement is the debonding of the FRP from the substrate, triggered by high stress concentrations at the extremities of the stiffener. A mixed-mode analysis [1–4], accounting for the normal stresses acting at the interface, is certainly the most accurate approach. However, considering the small thicknesses of the FRP strips, their bending strength can be neglected at least as a first order approximation, so that a pure mode II crack propagation can be assumed to describe the response of the bonded joint.

The debonding process is certainly complex and different experimental setups are used for its characterization (an extensive list of references can be found in [5,6]). In any case, there are a few objective parameters that characterize the ultimate performance. One of these is certainly the *Effective*-Bond-Length (EBL) of the stiffener, defined as the bond length beyond which no further increase of pull-out strength can be achieved. Knowledge of the EBL is necessary to properly design the reinforcement so to assure the complete transfer of load to the substrate.

To interpret the phenomenon of debonding, various shear-anchorage-strength models have been proposed, for which a review can be found in [6]. In general, such models can be classified into three categories: (i) empirical models based on the regression of test results [7]; (ii) engineering formulations based upon simplified assumptions and appropriate safety factors [8,6,9]; (iii) fracture-mechanics-based models [10–12]. All these aim at defining the pull-out-force vs. end-displacement curves [13,14] when the bond length is varied, from which the EBL can be determined.

To our knowledge, the major underlying assumption common to all the analytical approaches proposed so far consists in neglecting the elastic deformation of the substrate, so that the description of the entire phenomenon is deferred to the calibration of a proper shear-stress vs. slip interface constitutive law. But such approaches predict that the shear stress at the interface never reaches, but rather asymptotically approaches, the zero value. It is then difficult to objectively define the EBL, because the bond is active in the whole stiffener, whatever its length is.

This is why many researchers have given an engineering interpretation of the EBL. For example, many models define the EBL as the bond length over which the resultant of the shear contact stress is at least 97% of the ultimate strength[1] of an *infinite* stiffener [11,15–17]. According to other authors, the evaluation cannot but be purely experimental. Measuring the strain profile in the stiffener – usually employing resistance strain gages – the effective bond length is the length over which the strain decays

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[1] Notice, in passing, that a characteristic coefficient that appears in the governing equations relying upon the rigid-substrate hypothesis [11] is tanh 2, and $\tanh 2 \simeq 0.97$. Therefore, the limit value 97% is simply suggested by a mathematical formulation of the problem and is not justified on a physical basis.

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from the maximum to the zero value [13,18–22]. Empirical formulas can thus be proposed on the basis of the experimental results. Both definitions, however, carry intrinsic ambiguities. In the first case, the percentage is *a priori* defined, and the result is strongly dependent upon the assumed constitutive law at the interface. The second definition is affected by the experimental error and the sensitivity of the gages.

Another approximation associated with the assumption of rigid substrate is that the slip, i.e., the relative displacement between stiffener and substrate, is theoretically and experimentally evaluated by simply integrating the axial strain in the stiffener [23–25]. A more precise calculation would require the evaluation of the strain in the substrate, which is far from being negligible especially at those zones, like the stiffener extremities, where stress concentrations do occur. In any case, if the substrate is rigid the slip is always non-zero whatever the bond length is, regardless of the assumed constitutive law for the interface.

This paper continues and concludes a line of research by the authors where the effect of the elastic deformation of the substrate is assumed to play a significant role. The model problem now considered is that of an elastic stiffener in contact with the boundary of a semi-infinite plate, supposed in generalized plane stress. Problems of this kind in plane linear elasticity have been considered by various researchers [26–31], with the main purpose of evaluating the stress concentrations near the edges of the stiffener in relation with crack initiation and propagation in the substrate or along the interface.

In [32], the authors have solved the problem when the stiffener is pulled at one extremity (loaded end) and the bond is perfect (no slip occurs). An extension of Irwin's formula has been obtained to correlate the mode II stress intensity factor with the release of elastic strain energy associated with the detachment of the stringer. Assuming a Griffith-like energetic competition, debonding is assumed to start and develop when the energy release rate equals the surface energy of detachment. With this model the ultimate strength can be correctly predicted, but the EBL is strongly underestimated. In fact, the stress singularity at the loaded end predicted by the theory of elasticity equilibrates the greatest majority, by far, of the applied load. In other words, the calculated shear stress at the interface shows a decay much more rapid than in the experimental measurements.

To solve this inconsistency, in [14] a zone was supposed to exist in a neighborhood of the loaded end where cohesive forces *à là* Baranblatt may develop at the price of a relative slip between the adherents. Assuming a simple, step-wise, shear-stress vs. slip constitutive law at the interface, the length of the cohesive zone was evaluated by imposing that the stress intensity factor is null at the frontier with the perfectly adherent zone, in agreement with a procedure also followed in [33,34]. In this way, it was possible to demonstrate that the applied load is in practice equilibrated by the cohesive forces acting in the cohesive zone only. Therefore, the maximal length of the cohesive zone, compatible with the assumed constitutive interface law, provides a natural and physically motivated definition of the EBL. Experimental results confirm the analytical findings for various values of the bond length.

One major question is still open at this point, which regards the possible effects of the second (physically inconsistent) singularity acting at the free end of the stiffener. This is still present in the model of [14]. The major contribution of this paper is consideration of a second cohesive zone, governed by the same interface law used for the loaded end of the stiffener, able to annihilate also the singularity at the free end. Of course, the analysis complicates of one order of magnitude with respect to [14], because the two cohesive zones are not independent, but they influence one another. The problem is solved using a Chebyshev expansion that provides a complicated system of equations for the unknown coefficients. The formulations of [32,14] become particular limit cases of this more general and complete approach.

Effective material separation is supposed to start when the relative slip between the adherents exceeds a certain threshold. If the stiffener is sufficiently long, we show that there is maximal reachable length of the cohesive zone at the loaded end, which we will demonstrate to be coincident with the EBL.

Comparison of the three approaches (no cohesive zone, single cohesive-zone and double cohesive-zone) shows that, modulo proper calibration, all of them can predict the ultimate pull-out load in agreement with experiments, but only the cohesive models can accurately evaluate the EBL. More in particular, the state of stress at the free end of the reinforcement has a very little influence. In other words, the singularity at the free end of the stiffener carries a negligible part of the applied load. Therefore, the single cohesive-zone model results to be very accurate, but it avoids the noteworthy computational complications of the double cohesive-zone approach.

The cohesive models furnish values of the EBL in good agreement with experimental data recorded by the technical literature, evidencing the importance of cohesion forces in the analysis of a bonded joint. Comparisons with the formulas proposed by technical standards [35] evidence that such formulations, based upon the assumption of rigid substrate, tend in general to overestimate the EBL. To this respect, the proposed approach represents a substantial improvement.

## 2. Adhesion of an elastic stiffener to an elastic substrate

The contact problem of an elastic stiffener of finite length bonded to the boundary of an elastic semi-infinite plate and pulled at one end by a coaxial load is governed by a singular integral equation involving the unknown tangential contact forces [36]. If no slippage occurs between stiffener and plate, the theory of elasticity predicts that interface shear forces have a singularity at both ends of the stiffener. In order to remove this physical inconsistency, two cohesive zones are introduced at both edges of the reinforcement. The length of these zones depends upon the applied load, and can be found from condition that interface forces are finite in the whole bond, according to the same rationale followed by Barenblatt in the theory of cohesive cracks [37]. In Section 2.1 the resulting system of singular integral equations is solved through a Chebyshev expansion, while Sections 2.2 and 2.3 recover the solutions of one cohesive zone [14] and no cohesive zone (perfect bond) [32] as limit cases.

### 2.1. Double-Cohesive-Zone (DCZ) model

Consider an elastic stiffener of length $l$, thickness $t_s$ and constant width $b_s$, bonded to the boundary of an elastic semi-infinite plate in generalized plane stress of width $b_p$ (Fig. 1). At one

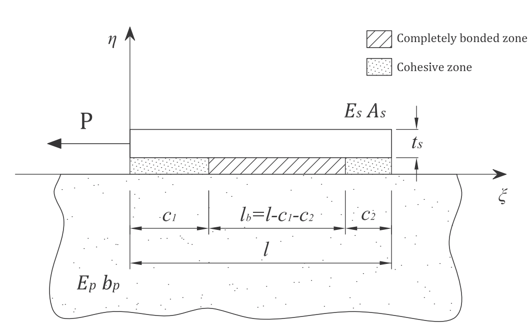


**Fig. 1.** A finite stiffener bonded to the boundary of a semi-infinite plate with cohesive zones at both ends.