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Modeling the dynamics of filaments for medical applications

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ABSTRACT

We present a theoretical formulation based on the Cosserat theory of rods to describe the motion of rod-like filaments (e.g., surgical threads) used in surgical suturing. The equations of motion are simplified using certain approximations and the resulting equations are solved using the finite difference (centered difference) scheme in time and space. The boundary conditions involve the specification of the force and moment at the free end of the thread and symmetry conditions commonly encountered during surgeries. Numerical simulations of suture threads with genuine material parameters and geometries specific to surgical scenarios are presented.

1. Introduction

Training in the task of surgical suturing is known to medical school students as being particularly challenging. For example, a novice trainee can take up to one minute to tie a surgical knot and, at this rate, the four hundred knots needed in a typical organ surgery would require six hours. On the other hand, traditional surgical training methods that rely on practicing on plastic/foam models, animals, corpse, or a combination of them have their limitations [1]. Among the frequently cited limitations are the differences between animal and human physiology, the variation of response between dead and healthy tissues, the increased risk of blood contamination for trainees during manipulation of sharp instruments, considerations of ethical nature for experiments on animals and conflicts from certain religious and social groups to practice on porcine tissues [2]. To address this problem, sporadic efforts have taken place over the last fifteen years to bring computer simulations into the training of medical procedures. Large US medical institutions (e.g., Massachusetts General Hospital, Stanford and Harvard medical schools, among others) now require training based on numerical simulations as part of their education [3]. Nonetheless, there remains much resistance and questions about their applicability. A chief reason for this seems to be the lack of realism rendered on screen which makes surgical scenarios simply too difficult to believe [4]. In addition, within these simulation efforts, modeling of tissues and organs [5–7] have received more attention than the modeling of surgical instruments [8] and suture threads [9–11]. Most often, a finite element analysis (FEM) is used to handle the complex geometries of organs (in particular brain and heart) but no such approach is completely satisfactory because meshing depends on the type of information needed. For example, a static mesh (i.e., a mesh that does not change with time) can be used for tissue closure while a dynamic mesh (i.e., a mesh changing with time) is needed to model tissue cutting [6]. In the case of suture thread, capturing the various configurations in space with a finite element analysis would require a very fine mesh and a large number of material parameters at each node and thus limiting the potential of being used for real-time simulations. In the literature, an interesting example of finite element analysis used to model suture threads is contained in the work of Berkley et al. [5]. In this work, the computational time is decreased by determining the output on a "need to know basis". The idea is that certain field variables need not be computed everywhere. For example, displacements and strains may not be needed at some interior points and determining the reaction forces may be useful only when the user touches the model. Another popular approach for modeling one-dimensional deformable structures is the spring-mass chain [9] or, similarly, cylinders connected by ball joints [12]. The shortcoming of these methods is that they are based on polynomial interpolation and not on a continuum model, and this limits the number of parameters that can be entered. In addition, these methods have been mostly employed to describe bending deformations. A departure from this is provided by Gregoire et al. [13] who monitored bending and twisting deformations with a spring-mass system. Their work follows experimental results of twisted rods by Goss et al. [14] and their subsequent loop formation. The results indicate that the experimental solution deviates from the analytical solution. Additional

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insights about the deviation between theory and experiments of rods are also contained in the work by Lakes [15].

The present study is concerned with the development of necessary physics-based numerical models to describe and simulate the behavior of surgical threads when subjected to conditions commonly encountered during surgery. This research is sponsored by Qatar national Research Fund (QNRF) and the research group consists of experts from the fields of mechanics, computer-graphics, and medicine. The immediate objective of the group is to develop a low-cost, user-friendly, interactive software to help medical school students practice the tasks of suturing in complement to the traditional training methods. We target medical school students for pedagogical benefits but also health professionals, novice or experts. The software to be developed could be used by nurses in clinics of remote areas to practice simple suturing tasks as well by expert surgeons to design new surgical procedures. The following discussion provides the technical background for the present study.

Surgical threads are long and thin circular structures (rod) with one dimension (namely, the length) much larger than the other dimensions. They can be modeled as a one-dimensional rod-like structures, which we refer here as filaments, while assuming a distinct three-dimensional coiling shape in space (Fig. 1). In the United States, suture threads available on the market are classified by United States Pharmacopeia (USP) as a function of diameter, length, and material properties, and which one is used depends on the nature of the surgery. Suture sizes vary from # 5 to #11-0, where # 5 represents a heavy braided suture typically used for orthopedics and # 11-0 corresponds to a fine monofilament suture used for ophthalmic. Size # 4 is roughly that of a tennis racket string. In addition to being non-toxic, typical requirements for suture threads are that they be strong enough not to break, flexible enough to be tied and knotted easily, and inextensible (i.e. that they do not stretch).

Among the choices for rod modeling, the most primitive is the Euler-Bernoulli beam theory [16] which, unlike the Timoshenko beam theory [16,17], does not account for the transverse shear strain effect. Instead, we favor the theory of rods introduced a century ago by the Cosserat brothers [18], just thirty years after the other popular Kirchhoff theory [19] (also referred as Kirchhoff–Clebsh theory). In both, the novelty resides in endowing a material point with six degrees of freedom. The difference between the two theories is that the Kirchhoff theory specifically assumes small displacements and strains while the Cosserat theory does not preclude large strains and displacements; in fact, there are no mentioning of constitutive equations (linear or not) at all in the work by the Cosserat so the equations are general enough to be applicable to solids and fluids. We note that with the assumption of material linearity in solids, both theories are entirely equivalent. The Cosserat theory has reemerged during the last twenty five years as having great potential to treat problems in three-dimensional elasticity. It has been used in diverse fields such as the automotive industry, the textile industry, marine engineering, bio-engineering, and more recently in computer graphics. Detailed description of this theory can be found in [20]. To summarize, the theory treats a material point as a "small" rigid body with six degrees of freedom corresponding to three displacements and three rotations. The rotation is the result of allowing gradient of forces between two opposite surfaces, unlike in the classical theory. In addition to the usual (contact) stresses, one has now to take into account couple-stresses, which are a measure of moment per unit area. A distinct feature of the three-dimensional system of Cosserat equations is that the (contact) stress tensor has non-symmetric components, unlike the classical theory, so eighteen components of (contact) stress are needed to fully describe the structure.

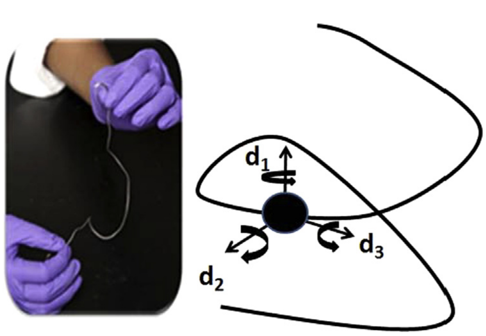

Fig. 1. Left, the surgical thread deforming in a three-dimensional space and (right) its idealization as a one dimensional Cosserat rod-like structure which treats each material point as a small rigid body with six degrees of freedom.

Specific to the description of a one-dimensional structure, the rod configuration is described by means of its centerline and its cross section. The centerline is a space curve parameterized by the arc length s and the orientation of the cross section passing through a point is described by means of three orthogonal vectors known as directors. The director normal to the cross section is usually denoted by $\boldsymbol{d}_3$ and it may be distinct from the tangent vector to the centerline. The work of Pai [11] has been one of the early attempts to model surgical threads using the Cosserat theory. The results can be extended to model other thin rod-like structures such as hair in the computer graphics or game industry. In it, algorithms available to solve rigid-body dynamics problems are being modified. This follows a powerful idea introduced by the Cosserat [18], which states that the configuration of the rod in space can be found by letting the arc length s play the role of time and be regarded as the trajectory just as in orbital mechanics or space probe problems. The equations solved are in their static form, and upon enforcing the two constraints of inextensibility and normality (i.e., the transverse shear strain is zero), the resulting equations are a system of ordinary differential equations (ODEs) with respect to space that can be solved as a boundary value problem (BVP). To capture the physics, where one end of the thread is attached to the skin and the other to the needle, a two-point BVP is solved. However, the system of equations is difficult to solve, and a stable solution is not as easy as with rigid-body dynamics problems although results are believable. However, the static model is not keen to accommodate the level of user-interaction with real time response needed. Goyal [21] solved the dynamic equations of filament-like DNA structures using an incremental rotation method which has no singularities unlike with the traditional Euler angles. The model can accommodate non-linear constitutive laws and capture non-homogeneous stiffness, kink formation, stress-free intrinsic curvature, and twist extension coupling [22]. In it, the dynamic solution is obtained with a generalized α-method in space and time, and it yields deformations matching those observed in reality.

Another restriction of the static solution is that it cannot be used for modeling knot tying. A number of knots simulations have emerged from the computer graphics industry in the last ten years. Knots are based on the mathematical theory of bifurcation [23] and simulating knots is difficult. A complete guide of knots used in surgery can be found in [24]. A knot is the result of complex crossing during which loops are formed. In the process of tying, some parts of the thread become in contact with other parts of the thread (self-collision) or collide with other objects (skin tissues). The model of Phillips et al. [25] is based on a spline on linear springs but it takes several minutes of computation to tie a simple knot, thus the algorithm is not time efficient. Lenoir et al. [10] used Lagrange multipliers to take into account the constraints during collision. The model of Wang et al. [26] on knot-tying is dynamic