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Axisymmetric static and dynamic buckling of hollow microspheres

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A B S T R A C T

Syntactic foams are particulate composites that are obtained by dispersing thin hollow inclusions in a matrix material. The wide spectrum of applications of these composites in naval and aerospace structures has fostered a multitude of theoretical, numerical, and experimental studies on the mechanical behavior of syntactic foams and their constituents. In this work, we study static and dynamic axisymmetric buckling of single hollow spherical particles modeled as non-linear thin shells. Specifically, we compare theoretical predictions obtained by using Donnell, Sanders–Koiter, and Teng–Hong non-linear shell theories. The equations of motion of the particle are obtained from Hamilton's principle, and the Galerkin method is used to formulate a tractable non-linear system of coupled ordinary differential equations. An iterative solution procedure based on the modified Newton–Raphson method is developed to estimate the critical static load of the microballoon, and alternative methodologies of reduced complexity are further discussed. For dynamic buckling analysis, a Newmark-type integration scheme is integrated with the modified Newton–Raphson method to evaluate the transient response of the shell. Results are specialized to glass particles, and a parametric study is conducted to investigate the effect of microballoon wall thickness on the predictions of the selected non-linear shell theories. Comparison with finite element predictions demonstrates that Sanders–Koiter theory provides accurate estimates of the static critical load for a wide set of particle wall thicknesses. On the other hand, Donnell and Teng–Hong theories should be considered valid only for very thin particles, with the latter theory generally providing better agreement with finite element findings due to its more complete kinematics. In this context, we also demonstrate that a full non-linear analysis is required when considering thicker shells, while simplified treatment can be utilized for thin particles. For dynamic buckling, we confirm the accuracy of Sanders–Koiter theory for all the considered particle thicknesses and of Teng–Hong and Donnell theories for very thin particles.

1. Introduction

Syntactic foams are a class of particulate composites that are fabricated by dispersing hollow microspheres, called microballoons, in a matrix material (see for example [49,67]). Microballoons are often produced from glass [38,72,71], carbon [17,18], or metal [7,9,22]. Such reinforcements provide closed-cell porosity, which, in turn, can be used to reduce the material density [10,77], improve the mechanical properties [11,28,30,47,54,83], and decrease thermal expansion and moisture absorbtion [62,74,78].

Syntactic foam design relies on the understanding of the failure mechanics of microballoons under static and dynamic loading conditions [12,17,18,38,45]. Through experimental efforts and finite element simulations, these studies have characterized the response of microballoons to uniaxial compressive loading. Specifically, for a microballoon compressed by two parallel flat plates, these works have elucidated the role of particle wall thickness [17,18], diameter [17,38], loading rate [45], and geometrical imperfections in the form of holes or nonuniform wall-thickness on microballoon deformation and failure [18].

To reduce computational complexity and offer insight into geometric non-linearities, recent efforts [36,37,64] have proposed shell-like models for microballoon deformations. These studies focus on the static response of a microballoon embedded in an infinitely extended elastic material, subject to remote uniaxial compression, to investigate the effect of matrix compliance on microballoon axisymmetric buckling. In Shams et al. [64], the shell is modeled using Donnell non-linear shell theory and a full non-linear buckling analysis is performed to estimate the critical compressive load; while Koiter non-linear shell formulation is adopted by Jones et al. [36] along with a simplified buckling analysis. Comparison between finite element results and theoretical findings on glass–vinyl ester systems demonstrates the accuracy of Donnell theory and the need for incorporating salient non-linearities in the estimation of the critical load for the

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axisymmetric buckling of a microballoon embedded in an infinitely extended matrix. However, not only is the accuracy of Donnell theory in studying microballoon buckling in the absence of the surrounding matrix currently untested, but also related findings by Amabili [3] on fluid-filled and empty circular cylinders suggest that Donnell theory's predictions lose accuracy in the absence of the stabilizing effect of an interacting medium. The main objective of the present study is to critically compare different spherical cap non-linear shell theories and simplified analysis procedures to address static and dynamic buckling of isolated microballoons under uniaxial compressive loading.

Several non-linear shell models have been proposed in the literature to describe thin-walled structures under static or dynamic loading, such as the classical theories by Donnell [23], Sanders–Koiter [61], Novozhilov [52], Flügge–Lur'e–Byrne [25], and Rotter and Jumikis [59], along with the more recent theory by Teng and Hong [75], which retains all non-linear terms in the strain–displacement relation from three-dimensional elasticity. In [3], Donnell, Sanders–Koiter, Novozhilov, and Flügge–Lur'e–Byrne theories are employed to study large amplitude vibration of both fluid-filled and empty circular cylinder shell. Sahu and Datta [60] and Qatu [56] have conducted extensive reviews on the dynamic instability and transient response of shells, respectively, in terms of different geometries, loading conditions, shell theories, analysis methods, and boundary conditions. While such theories are applicable to the analysis of static and dynamic instabilities of a wide set of shell geometries, most research efforts focus on cylindrical (see for example [6,41]) and shallow spherical shells (see for example [4,24,31,50,80]).

In this work, we consider the implementation of Donnell, Sanders–Koiter, and Teng–Hong theories to study axisymmetric static and dynamic buckling of a microballoon. We present a unified framework for the selected theories on the basis of the comprehensive non-linear theory reported by Teng and Hong [75]. We derive the equations of motion of the spherical shell by employing Hamilton's principle. The Galerkin projection technique is adapted to compute a solution for the non-linear system of partial differential equations governing the shell motion. An iterative scheme based on the modified Newton–Raphson method is used to compute the spherical shell displacement for static loading. For the analysis of dynamic buckling, we couple a Newmark-type time integration with the modified Newton–Raphson iteration scheme to study the transient response of the shell. The static and dynamic instabilities of the shell are identified by analyzing the eigenvalues of the tangent stiffness matrix and the time trace of the displacement of the shell, respectively. Further, for static buckling, we investigate two approximate methods to evaluate the critical load that do not require tracking of the shell non-linear deformations prior to buckling. In one method, we simply utilize a linear model for the shell deformation until the onset of buckling. In the other approach, we adapt the approximation by Jones et al. [36], whereby the tangent stiffness matrix is computed by neglecting non-linearities in the constitutive behavior of the membrane resultants. A parametric study is performed to elucidate the influence of the shell thickness on the different non-linear theories. Findings from the static and dynamic buckling are validated through finite element simulations.

The paper is organized as follows. In Section 2, we state the problem and introduce the non-linear kinematic assumptions for the considered shell theories. In Section 3, we derive the non-linear equations of motion from Hamilton's principle. In Section 4, the solution procedure for finding the instability point of the system for static and dynamic buckling is described. In Section 5, we specialize our analysis to glass microballoons and perform a parametric study to assess the accuracy of the non-linear shell theories for static and dynamic buckling against finite element findings. Conclusions are reported in Section 6. Further, in Appendix A, we present the complete expressions for the coefficients introduced in Section 4. Details of the implementation of the finite element analysis and a mesh independence study are discussed in Appendix B. A convergence study for the Galerkin procedure is discussed in Appendix C.

2. Problem statement

The geometry of the spherical shell is described through the mean radius R and the thickness h. We focus on thin shells for which the thickness-to-radius ratio h/R is on the order of 0.01. A spherical coordinate system (r,θ,ϕ) is used to describe the shell mid-surface, where r is the radial coordinate, θ is the meridional direction (zenith angle), and ϕ is the circumferential direction (azimuth angle). Throughout this work, the equator and north pole of the sphere are defined as the points at $y=0$ and R, respectively, that is, at $\theta=\pi/2$ and 0, see Fig. 1. The shell material is assumed to be linear, elastic, isotropic, and homogenous. We hypothesize that the shell is subject to a traction field that is axisymmetric deformations with respect to the y-axis and results in axisymmetric deformations. Therefore, the displacement component along the azimuth direction is zero and the radial and meridional displacement fields are independent of ϕ. Moreover, the ϕ direction is a principal stress axis. Thus, resting on the assumption of axisymmetric shell deformation, the original three-dimensional problem reduces to a two-dimensional one. We note that numerical and experimental studies on the buckling of microballoons used in syntactic foam fabrication [17,18,45] support the accuracy of this assumption for uniaxial loading.

Based on Love's hypothesis (see for example [5,68]), the strain components for the thin shells read

$$\varepsilon_{\theta\theta}(r,\theta,\phi,t)=\varepsilon_{\theta\theta,0}(\theta,\phi,t)+\zeta\kappa_{\theta\theta}(\theta,\phi,t) \tag{1a}$$

$$\varepsilon_{\phi\phi}(r,\theta,\phi,t)=\varepsilon_{\phi\phi,0}(\theta,\phi,t)+\zeta\kappa_{\phi\phi}(\theta,\phi,t) \tag{1b}$$

$$\varepsilon_{\theta\phi}(r,\theta,\phi,t)=\varepsilon_{\theta\phi,0}(\theta,\phi,t)+\zeta\kappa_{\theta\phi}(\theta,\phi,t) \tag{1c}$$

where ζ represents the distance of the any arbitrary point of the shell from the mid-surface, that is, $\zeta=r-R\in[-h/2,h/2]$; $\varepsilon_{\theta\theta,0}$, $\varepsilon_{\phi\phi,0}$, and $\varepsilon_{\theta\phi,0}$ are the mid-surface strains; $\kappa_{\theta\theta}$, $\kappa_{\phi\phi}$, and $\kappa_{\theta\phi}$ are the mid-surface changes in curvature; and t is the time variable. Due to the symmetry of the problem, the radial and meridional displacement fields are independent of the azimuth angle ϕ; thus, the shell strain fields are functions of only θ and t and both $\varepsilon_{\theta\phi,0}$ and $\kappa_{\theta\phi}$ are zero. For convenience, we non-dimensionalize the displacement fields with respect to the shell radius R, so that w

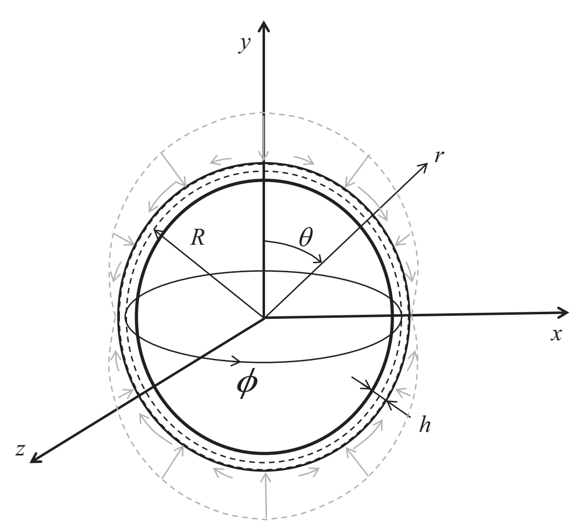

Fig. 1. Schematic depiction of the geometry, nomenclature, and traction fields.