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Nonlinear primary resonance of nano beam with axial initial load by nonlocal continuum theory

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ABSTRACT

Based on the nonlocal continuum theory, the nonlinear primary resonance of nano beam with the axial initial load is investigated. The amplitude–frequency response for the primary resonance is derived with the multiple scale method and the stability is analyzed. The nonlinear primary resonance of nano beam is discussed with the influences of small scale effect, axial initial load, mode number, Winkler foundation modulus and the ratio of the length to the diameter. From the results, the typical hardening nonlinearity can be observed. Moreover, some significant and interesting nonlinear phenomena can be found for the primary resonance of nano beam. This work is expected to be useful for the design and analysis for the nonlinear dynamic behaviors of structures at nano scales.

1. Introduction

With the superior mechanical characteristics for nano structures, microelectromechanical (MEMS) and nanoelectromechanical (NEMS) devices are widely applied [1–4]. As the typical element in the nano mechanical systems, nano beam can be used as nano sensors, actuators, molecular bearings and atomic-force microscope [5–8]. Therefore, both static and dynamic behaviors of nano beam have drawn a lot of attention and shown the size-dependent properties [9–13]. Because it is difficult to control experiments at the nanometer scale, proper and effective mathematical methods have been performed.

The molecular dynamics (MD) simulation is rather difficult to be carried out with large-scale nano systems and limited to a small case. Although the classical continuum method provides simple mechanical models and possesses the characteristic of less time-consuming, it cannot present the size-dependent behaviors and lacks the accurate description of nano systems. It should be noted that these characteristics become more significant at the small scales for nano beams [14–16].

The nonlocal continuum theory presented by Eringen [17,18] is different from the classical models. Because it assumes that the stress at a reference point behaves as a function of the strain at every point in the body, it can provide proper and reliable results. As a result, based on the nonlocal model, many researches have been reported on the buckling [9,12,13,19,20], vibration [21–23] and wave propagation [24,25] characteristics of nano beams, in which the scale effects are considered. In recent years, besides a lot of research articles have been reported on the nano scale structures by the nonlocal continuum theory, more explicit information can be found in some review papers [26–28].

From the research status of dynamical properties for nano beams, it is found that only several papers have been reported on nonlinear vibration properties by both classical and nonlocal continuum theories. As an early work on the nonlinear vibration of nano beams/nanotubes, Fu et al. studied the nonlinear free vibration of embedded nanotubes with the classical beam model and the incremental harmonic balanced method [29]. Ke et al. presented the nonlinear free vibration of embedded double-walled nanotubes with the nonlocal Timoshenko beam theory [30]. Reddy gave the nonlocal nonlinear formulations of classical and shear deformation theories for beams and plates [31]. By the classical Euler–Bernoulli beam model, Khorasany and Hutton analyzed the wave propagation characteristics of nanotubes with the temperature and geometrical nonlinearity [32]. Ansari and Hemmatnezhad applied the finite element formation to present the frequency response of nonlinear free vibration for double-walled nanotubes [33]. Soltani and Farshidianfar presented the periodic solution for nonlinear free vibration of nanotube conveying fluid [34].

Moreover, due to the large elastic deformation, nonlinear vibration for the microelectromechanical (MEMS) and nanoelectromechanical

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(NEMS) always appears in practical engineering. Nonlinear vibration characteristics play an important role on the design and analysis of MEMS and NEMS, in which the methods are quite different from the linear vibration. The primary resonance of MEMS and NEMS has been reported by Kacem et al. [35]. As a series work, Kacem et al. used the classical continuum method for nonlinear dynamics of MEMS and NEMS and derived some interesting and valuable results [35–39].

Based on these papers, it can be seen that large deformation within the elastic limit and nonlinear behaviors are usually observed for nano beams. As a result, the finite deformation makes the nonlinear analysis essential. Although most dynamical researches are concerned on the linear properties, nonlinear analyses are necessary as one of the interesting and valuable directions in future studies on nano beams. However, few investigations have been presented on the nonlinear dynamic properties of nano beam with the external harmonic excitation or axial initial load, especially for the resonant characteristics with small scale effects. Taking these factors into account, proper understanding and development of nonlinear vibration properties of nano beams can provide a useful help for the design and analysis of MEMS/NEMS devices working at large amplitudes.

In this work, the primary resonance of nano beam with the axial initial load is concerned and studied by the nonlocal continuum theory. Both damping and small scale effects are considered. The influences of the scale coefficient, mode order, initial load, ratio of the length to the diameter and Winkler foundation modulus of elastic matrix on the frequency response of the primary resonance are analyzed.

2. Equation of nonlinear vibration

The nano beam embedded in the viscous elastic matrix is shown in Fig. 1. According to the work of Eringen [17,18], the constitutive relation of nonlocal elasticity is presented with the form of the integral equation as

$$\sigma_{kl,k} - \rho\, \ddot{u}_l = 0, \tag{1a}$$

$$\sigma_{kl}(\mathbf{x}) = \int_V \alpha(\mathbf{x},\mathbf{x}')\tau_{kl}(\mathbf{x}')dV(\mathbf{x}'), \tag{1b}$$

$$\varepsilon_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}), \tag{1c}$$

where σ_{kl} is the nonlocal stress tensor, ε_{kl} the strain tensor, ρ the mass density, u_l the displacement vector, $\tau_{kl}(\mathbf{x}')$ the classical (i.e. local) stress tensor, $\alpha(\mathbf{x},\mathbf{x}')$ the kernel function which describes the influence of the strains at various location $\mathbf{x}'$ on the stress at a given location $\mathbf{x}$ and V the entire body considered.

We can observe from Eq. (1) that not only the strain state of the reference location $\mathbf{x}$ has the influence on the stress state at $\mathbf{x}$, but also the strain state at $\mathbf{x}'$ can affect on the stress state of the same location. Then, the following relation can be derived:

$$[1-(e_0a)^2\nabla^2]\boldsymbol{\sigma} = \mathbf{C}_0 : \boldsymbol{\varepsilon}, \tag{2}$$

where $\mathbf{C}_0$ is the elastic stiffness matrix of classical elasticity, e_0 the constant appropriate to each material and a the internal characteristic length (e.g., the length of C–C bond, the lattice spacing and the granular distance). It should be noted that the value of e_0 is determined from experiments or by matching dispersion curves of the plane waves with the atomic lattice dynamics. So e_0a means the scale coefficient which denotes the small scale effect on the mechanical characteristics of nano structures. It will be reduced to the classical (i.e. local) model for $e_0a=0$.

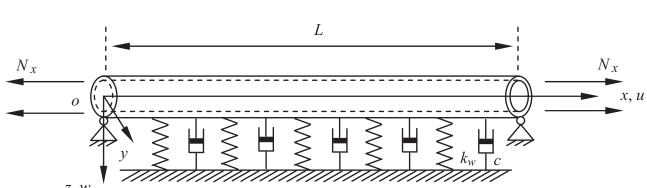

Fig. 1. Nano beam embedded in viscous elastic matrix with axial initial load.

For the Euler–Bernoulli beam model, the axial force and the resultant bending moment are

$$N = \int_A \sigma_x dA, \quad M = \int_A z\sigma_x dA, \tag{3}$$

where z is the transverse coordinate measured in the direction of the deflection and A the area of the cross section of the nano beam.

The displacement fields can be expressed as the following form:

$$u_1(x,z,t) = u(x,t) - z\frac{\partial w}{\partial x}, \quad u_2 = 0, \quad u_3(x,z,t) = w(x,t), \tag{4}$$

where u and w are the axial and transverse displacements, respectively.

For the nonlinear vibration by the large amplitude, the nonzero von Kármán nonlinear strain should be considered and the relation between the strain and displacement is

$$\varepsilon_0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_1 = -z\kappa \tag{5}$$

where ε_0 is the nonlinear extensional strain, $\kappa = -\partial^2 w/\partial x^2$ the bending strain and ε_1 the strain induced by κ. Then, the von Kármán nonlinear strain (i.e. ε_{non}) is

$$\varepsilon_{non} = \varepsilon_0 + \varepsilon_1 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2}, \tag{6}$$

For the transverse vibration with large deformation, the vibration equation can be expressed as [40,41]

$$\frac{\partial S}{\partial x} = k_w w + c\frac{\partial w}{\partial t} + \rho A\frac{\partial^2 w}{\partial t^2} - N_x\frac{\partial^2 w}{\partial x^2}, \tag{7}$$

where $S = \partial M/\partial x$ is the shear force, k_w the Winkler foundation modulus which can be described as the Winkler model [42,43], c the damping coefficient, N_x the axial load which can be expressed as $N_x = \sigma_x^0 A$ and σ_x^0 the axial initial stress. Usually, the dimensionless axial stress $\beta = \sigma_x^0/E$ is used. A negative β denotes the compression load and a positive β means the tension case.

From Eqs. (2)–(5), we have the following relation:

$$N - (e_0a)^2\frac{\partial^2 N}{\partial x^2} = EA\varepsilon_0, \tag{8a}$$

$$M - (e_0a)^2\frac{\partial^2 M}{\partial x^2} = EI\kappa, \tag{8b}$$

where E the Young's modulus and $I = \int_A z^2 dA$ the moment of inertia.

As a result, for the nano beam embedded in elastic matrix under the transverse harmonic excitation, the nonlinear vibration equation can be derived as

$$EI\frac{\partial^4 w}{\partial x^4} + \left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]\rho A\frac{\partial^2 w}{\partial t^2} + \left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]k_w w + \left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]c\frac{\partial w}{\partial t}$$

$$= \left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]N_0\frac{\partial^2 w}{\partial x^2} + \left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]$$

$$\left[\frac{EA}{2L}\int_0^L\left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2} + \left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]f\cos\Omega t, \tag{9}$$

where f is the spatial distribution of the transverse load and Ω the corresponding frequency.