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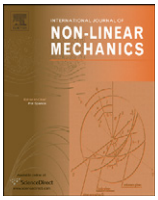

Analysis of a benchmark solution for non-Newtonian radial displacement in porous media

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ABSTRACT

We present an analytical formulation useful to interpret the key phenomena involved in non-Newtonian displacement in porous media and an analysis of the results obtained by considering the uncertainty associated with relevant problem parameters. To derive a benchmark solution, we consider the radial dynamics of a moving stable interface in a porous domain saturated by two fluids, displacing and displaced, both non-Newtonian of shear-thinning power-law behavior, assuming the pressure and velocity to be continuous at the interface, and constant initial pressure. The flow law for both fluids is a modified Darcy's law. Coupling the nonlinear flow law with the continuity equation, and taking into account compressibility effects, yields a set of nonlinear second-order partial differential equations. Considering two fluids with the same flow behavior index n allows transformation of the latter equations via a self-similar variable; further transformation of the equations incorporating the conditions at the interface shows for $n<1$ the existence of a compression front ahead of the moving interface. Solving the resulting set of nonlinear equations yields the positions of the moving interface and compression front, and the pressure distributions; the latter are derived in closed form for any value of n. A sensitivity analysis of the model responses is conducted both in a deterministic and a stochastic framework. In the latter case, Global Sensitivity Analysis (GSA) of the benchmark analytical model is adopted to study how the effects of uncertainty affecting selected parameters: (a) the fluids flow behavior index, (b) the relative total compressibility and mobility in the displaced and displacing fluid domains, and (c) the domain permeability and porosity, propagate to state variables. The relative influence of input parameters on model outputs is evaluated by means of associated Sobol indices, calculated via the Polynomial Chaos Expansion (PCE) technique. The goodness of the results obtained by the PCE is assessed by comparison against a traditional Monte Carlo (MC) approach.

1. Introduction

Displacement phenomena in porous media involving non-Newtonian fluid behavior are of considerable interest in several areas of engineering and physics. In petroleum engineering, various substances injected into underground reservoirs to enhance oil recovery, by improving the overall sweeping efficiency and minimizing instability effects, reveal a nonlinear stress-shear rate relationship and other non-linear effects [1]: these include dilute polymer solutions, emulsions of surfactants and foams [2,3]. On the other hand, heavy and waxy oils are often found to exhibit non-Newtonian characteristics at reservoir conditions [4,5]; therefore a situation may be envisaged in which a non-Newtonian fluid injected into a reservoir displaces another non-Newtonian fluid with different rheological characteristics. A similar situation may arise in environmental remediation efforts geared towards in situ treatment, where injection of substances having nonlinear rheological properties such as colloidal or biopolymer suspensions is employed to remove, or favor the removal of, liquid pollutants from contaminated soils; relevant examples include DNAPLs remediation by means of colloidal liquid aphrons [6], and the use of xanthan gum to enhance mobility and stability of suspensions of nanoscale iron employed in reactive barriers [7]. As in situ bioremediation may create polymers with non-Newtonian characteristics [8], a subsequent injection may result in displacement of a non-Newtonian fluid by another. Similar situations may arise in industrial engineering, where non-Newtonian flows occur in filtration of polymer melts, food processing, and fermentation [9], and in orthopedic applications, where injectable cements used in a variety of bone augmentation and bone reconstruction procedures also display a complex rheology [10].

The displacement phenomenon of a fluid by another in a porous domain has been extensively investigated in the literature when either fluid, or both, exhibit non-Newtonian behavior.

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Pascal [11] adopted Muskat's frontal advance model to study steady-state immiscible displacement of a Bingham fluid by another in plane/radial geometry. Steady-state displacement, and its stability, were analyzed in Ref. [12] for power-law fluids with yield stress in plane geometry, and in Ref. [13] for power-law fluids in radial geometry; capillarity was added to the model in Ref. [14]. In Ref. [15], transient plane displacement of a power-law compressible fluid by another was considered. In Refs. [16] and [17], transient plane/radial displacement of a power-law fluid by another was considered, allowing for two-phase flow behind the displacement front but neglecting compressibility. An analytical solution for piston-like displacement of power-law dilatant fluids in plane and radial geometry was derived in Ref. [18]. In Wu et al. [19] an analytical solution of Buckley–Leverett type to two-phase flow determined by the displacement of a Newtonian fluid by a non-Newtonian power-law one was obtained and validated by a numerical model. Wu and Pruess [20] developed a numerical simulator for multiphase flow in porous media, including the power-law and Bingham models. A novel two-phase numerical simulator incorporating non-Newtonian behavior was proposed in Ref. [21]. Tsakiroglou [22] generalized the macroscopic equations of the two-phase flow in porous media accounting for capillarity for the case of a shear-thinning displacing fluid, and developed a numerical scheme of inverse modeling to estimate model parameters from unsteady-state experiments. Other researchers investigated the onset of instabilities in displacement of non-Newtonian fluids experimentally [22–25] or theoretically [26].

If a fingering instability does not develop at the interface between displacing and displaced fluid, the frontal advance theory may be considered an approximate yet acceptable description of the displacement mechanism, with the advantage of providing analytical solutions, which in turn may prove useful as benchmarks against which numerical solvers are tested.

An example of such solutions was provided by Ref. [15], who derived a similarity solution for planar transient immiscible displacement of a power-law compressible fluid by another with the same flow behavior index. The study of the radial case (e.g., flow away from a wellbore), which represents a plausible simplification of the geometry involved in several possible applications, is developed in this work. The assumption of identical flow behavior index for displacing and displaced fluid is retained to derive a closed-form solution in the format of a system of algebraic nonlinear equations. As values on flow behavior index in real applications, especially connected to reservoir engineering, tend to cluster around 0.6–0.8 [27], the proposed solution may provide a qualitative insight on relevant physical phenomena also for fluids whose flow behavior index differ to some extent. The problem is formulated in dimensionless form for different types of boundary conditions in the origin of the flow domain (assigned pressure or flow rate), and novel closed-form expressions of the pressure field in the displacing and displaced fluids for a generic value of the flow law exponent are derived generalizing to two fluids the results of [28]; a discussion of deterministic results is then provided.

Uncertainty plagues virtually every effort to predict the behaviour of complex physical systems; in the problem under investigation, it affects to various degrees: (a) the properties of the porous medium, due to its inherent spatial heterogeneity and lack of complete characterization; (b) the descriptive parameters of the fluids involved, having a complex rheological behavior. In the first case, a random field description e.g., [29] represents the most complete methodology. In the sequel, to exemplify our approach and achieve easily interpretable indications, we model key problem parameters as independent random variables having an assigned probability distribution.

Global Sensitivity Analysis (GSA), conducted by computing Sobol indices as sensitivity measure (since no assumptions of linearity or monotonic behavior on model equations are required) [30,31], is a powerful instrument to investigate the relative influence of the different sources of uncertainty on the state variables of interest and represents the basis for a rational design of a measurement strategy in contaminant transport in porous media [32]. A reliable technique for the evaluation of Sobol indices is constituted by the Polynomial Chaos Expansion (PCE) technique, introduced in engineering context inside the stochastic finite elements analysis (SFEM) [33]. This method returns accurate results and drastically decreases the computational cost associated with GSA, otherwise unaffordable especially for complex numerical models [34].

In this work, we adopt GSA conducted by means of PCE to study how uncertainty affecting selected parameters: (a) the fluids flow behavior index, (b) the relative total compressibility and mobility in the displaced and displacing fluid domains, and (c) the domain permeability and porosity, propagates to state variables adopting the benchmark analytical model of non-Newtonian radial displacement derived earlier. The goodness of the results obtained by the PCE is then assessed by comparison against a traditional Monte Carlo (MC) approach.

2. Analytical model of non-Newtonian displacement and similarity solution

2.1. Flow law for power-law fluid in a porous medium

Flow of Newtonian fluids in porous media is governed by Darcy's law. Its extension to non-Newtonian fluids is complex, due to interactions between the microstructure of porous media and the rheology of the fluid, even in the creeping flow regime. The scientific literature of the past decades includes numerous works dedicated to this problem: for exhaustive reviews see Refs. [9,35,36]. A sizable part of them deals with power-law fluids, described by the rheological Ostwald–DeWaele model, given for simple shear flow by

$$\tau = m\dot{\gamma}|\dot{\gamma}|^{n-1}, \tag{1}$$

where τ is the shear stress, $\dot{\gamma}$ the shear rate, $m[ML^{-1}T^{n-2}]$ and n indices of fluid consistency and flow behavior respectively, with $n<1$, $=1$ or >1 describing respectively pseudoplastic, Newtonian, or dilatant behavior. The power-law model, itself a simplification of more complex, and realistic, rheological behavior, is nevertheless often adopted in both porous media and free-surface flow modeling for its simplicity [37]. The corresponding modified version of Darcy's law takes in the literature the two equivalent forms [35,38–44]

$$\nabla P = -\frac{\mu_{ef}}{k}|\mathbf{v}|^{n-1}\mathbf{v} = -\frac{m}{k^*}|\mathbf{v}|^{n-1}\mathbf{v}, \tag{2}$$

where $P=p+\rho gz$ is the generalized pressure, p the pressure, z the vertical coordinate, ρ the fluid density, g the specific gravity, $\mathbf{v}$ the Darcy flux, k the intrinsic permeability coefficient $[L^2]$, μ_{ef} the effective viscosity $[ML^{-n}T^{n-2}]$, k^* the generalized permeability $[L^{n+1}]$; the ratio k/μ_{ef}, termed mobility, is given by Ref. [45]

$$\frac{k}{\mu_{ef}} = \frac{1}{2m}\left(\frac{n\phi}{3n+1}\right)^n\left(\frac{8k}{\phi}\right)^{(1+n)/2}. \tag{3}$$

where ϕ denotes the porosity. For $n=1$, the effective viscosity μ_{ef} reduces to conventional viscosity μ, and Eq. (2) reduces to Darcy's law $\nabla P = -(\mu/k)\mathbf{v}$. Earlier literature reviews, e.g. Ref. [27] demonstrate that the bulk of applications to non-Newtonian flows in