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New analytical results of energy-based swing-up control for the Pendubot

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ABSTRACT

In this paper, we revisit the energy-based swing-up control solutions for the Pendubot, a two-link underactuated planar robot with a single actuator at the base joint. The control objective is to swing the Pendubot up to its unstable equilibrium point (at which two links are in the upright position). We improve the previous energy-based control solutions by analyzing the motion of the Pendubot further. Our main contributions are threefold. First, we provide a bigger control parameter region for achieving the control objective. Specifically, we present a necessary and sufficient condition for avoiding the singular points in the control law. We obtain a necessary and sufficient condition on the control parameter such that the up–down equilibrium point (at which links 1 and 2 are in the upright and downward positions, respectively) is the only undesired closed-loop equilibrium point. Second, we prove that the up–down equilibrium point is a saddle via an elementary proof by using the Routh–Hurwitz criterion to show that the Jacobian matrix valued at the point has two and two eigenvalues in the open left- and right-half planes, respectively. We show that the Pendubot will eventually enter the basin of attraction of any stabilizing controller for all initial conditions with the exception of a set of Lebesgue measure zero provided that these improved conditions on the control parameters are satisfied. Third, we clarify the relationship between the swing-up controller designed via the partial feedback linearization and that designed by the energy-based approach. We present the simulation results for validation of these results.

1. Introduction

Considerable research activity has been received in the study of the control of underactuated mechanical systems which possess fewer actuators than degrees of freedom, see e.g., [1–12]. The design of such mechanisms that can perform complex tasks with less actuators allows to reduce cost and weight by using fewer actuators and permits to increase fault tolerance to actuator failure. However, controlling these systems is challenging due to complex nonlinear dynamics, nonholonomic behavior, and lack of linearizability exhibited by these systems.

The Pendubot is a two-link planar robot with a single actuator at the base joint of the first link, and the joint between two links is unactuated and allowed to swing freely [2]. It, together with other mechanical systems such as the inverted pendulum on a cart [4], the Acrobot [1,13], is used for control and robot education and for research as one of typical examples of underactuated mechanical systems.

Many researchers study the swing-up control of pendulum-type systems, e.g., [1,2,4,13,14]. The swing-up control problem for the Pendubot is to swing the Pendubot up to its unstable upright equilibrium point (at which two links are in the upright position), and then balance it at that point [2]. For solving such a problem, Spong and Block [2] used partial feedback linearization approach for the swing-up control and used a LQR (linear quadratic regulator) for the balancing control. However, no stability analysis was provided there. Moreover, since there is no guarantee of entering the basin of attraction of the LQR controller, it is difficult to tune the control parameters to accomplish a successful switch from the swing-up phase to the balancing phase.

To overcome such a difficulty, Fantoni et al. [15] showed that if the total mechanical energy can be controlled to the potential energy at the upright equilibrium point and the first link can be stabilized to the upright positive, then the second link moves clockwise or counter-clockwise until it reaches the upright position according to a homoclinic orbit; this shows that the Pendubot can be swung up to an arbitrarily small neighborhood of the upright equilibrium point. This guarantees that the switch between the two phases can be easily accomplished. For achieving the above control objective, Fantoni et al. [15] reported an energy-based control solution to the swing-up control problem of the Pendubot where some conditions on the initial state of

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the Pendubot are required. Kolesnichenko and Shiriaev [16] studied the partial stabilization of a class of underactuated Euler–Lagrange systems by controlling the total mechanical energy and the actuated variables; and showed its application to the swing-up control problem of the Pendubot.

In this paper, we revisit the seminal energy-based control solutions for the Pendubot in [15,16]. In comparison with these papers, our main contributions are threefold. First, we provide a bigger control parameter region for achieving the control objective, which gives us more freedom to tune the control parameters for achieving a better control performance such as swinging up the robot quickly into a small neighborhood of the upright equilibrium point. Indeed, we present a necessary and sufficient condition for avoiding the singular points in the control law rather than the sufficient conditions in [15,16]. It was shown in [16] that the Pendubot can be swung up to an arbitrarily small neighborhood of the upright equilibrium point, or remain at the up–down equilibrium point (at which links 1 and 2 are in the upright and downward positions, respectively) provided that a condition on the control parameter related to the angle of the first link is satisfied. For a further improvement, we obtain a necessary and sufficient condition on the control parameter.

Second, we prove that the up–down equilibrium point is a saddle (hyperbolic and unstable). Indeed, we present an elementary proof by using the Routh–Hurwitz criterion to show that the Jacobian matrix valued at the point has two and two eigenvalues in the open left- and right-half planes, respectively. Thus, we prove that the Pendubot will eventually enter the basin of attraction of any stabilizing controller for all initial conditions with the exception of a set of Lebesgue measure zero provided that these improved conditions on the control parameters are satisfied.

Third, we clarify the relationship between the two controllers designed respectively by the partial feedback linearization and the energy-based approach, and present new simulation results for the Pendubot manufactured by Mechatronics Systems Inc. [17].

The remainder of the paper is organized as follows: In Section 2, some preliminary knowledge about two-link robots is recalled. In Section 3, an energy-based swing-up controller for the Pendubot is examined. The global motion analysis of the Pendbot is carried out in Section 4. Discussion is presented in Section 5. Simulation results are reported in Section 6 and conclusion is made in Section 7.

2. Preliminary knowledge

Consider the Pendubot, a two-link planar revolute robot with the second joint being passive shown in Fig. 1. For the ith ($i=1,2$) link, m_i is its mass, l_i is its length, l_{ci} is the distance from joint i to its center of mass (COM), and J_i is the moment of inertia around its COM.

Let $q=[q_1,q_2]^{\mathrm{T}}$ be the vector of the angles of two joints. In this paper, the angle of active joint, q_1, is dealt with in $\mathbb{R}$; the angle of the passive joint, q_2, is dealt with in $\mathbb{S}$ (a unit circle). The motion equation of the Pendubot is

$$M(q)\ddot{q}+H(q,\dot{q})+G(q)=B\tau_1, \tag{1}$$

where

$$M(q)=\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22}\end{bmatrix}=\begin{bmatrix} \alpha_1+\alpha_2+2\alpha_3\cos q_2 & \alpha_2+\alpha_3\cos q_2 \\ \alpha_2+\alpha_3\cos q_2 & \alpha_2\end{bmatrix}, \tag{2}$$

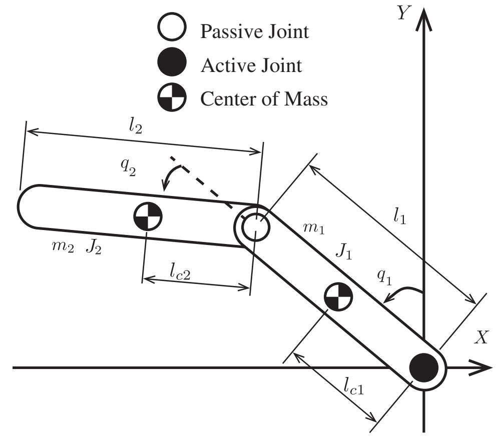

Fig. 1. The Pendubot: a two-link robot with a passive second joint.

$$H(q,\dot{q})=\begin{bmatrix} H_1 \\ H_2\end{bmatrix}=\alpha_3\begin{bmatrix} -2\dot{q}_1\dot{q}_2-\dot{q}_2^2 \\ \dot{q}_1^2\end{bmatrix}\sin q_2, \tag{3}$$

$$G(q)=\begin{bmatrix} G_1 \\ G_2\end{bmatrix}=\begin{bmatrix} -\beta_1\sin q_1-\beta_2\sin(q_1+q_2) \\ -\beta_2\sin(q_1+q_2)\end{bmatrix}, \tag{4}$$

B is the input matrix of

$$B=\begin{bmatrix} 1 \\ 0\end{bmatrix}, \tag{5}$$

τ_1 is the torque applied to joint 1,

$$\begin{cases} \alpha_1=m_1l_{c1}^2+m_2l_1^2+J_1, \\ \alpha_2=m_2l_{c2}^2+J_2, \quad \alpha_3=m_2l_1l_{c2}, \\ \beta_1=(m_1l_{c1}+m_2l_1)g, \quad \beta_2=m_2l_{c2}g, \end{cases} \tag{6}$$

and g is the acceleration of gravity.

The energy of the Pendubot is expressed as

$$E(q,\dot{q})=\frac{1}{2}\dot{q}^{\mathrm{T}}M(q)\dot{q}+P(q), \tag{7}$$

where $P(q)$ is the potential energy and is defined as

$$P(q)=\beta_1\cos q_1+\beta_2\cos(q_1+q_2). \tag{8}$$

3. Swing-up controller for the pendubot

We recall the energy-based swing-up controller in [15,16]. The new result of this section is the presence of a necessary and sufficient condition for nonexistence of any singular point in the controller.

Consider the following upright equilibrium point of the Pendubot:

$$q_1=0, \quad q_2=0 \ (\mathrm{mod}\ 2\pi), \quad \dot{q}_1=0, \quad \dot{q}_2=0. \tag{9}$$

For $E(q,\dot{q})$, $\dot{q}_1$, and q_1, if we can design τ_1 such that

$$\lim_{t\to\infty}E(q,\dot{q})=E_r, \quad \lim_{t\to\infty}\dot{q}_1=0, \quad \lim_{t\to\infty}q_1=0, \tag{10}$$

where

$$E_r=\beta_1+\beta_2 \tag{11}$$

is the potential energy of the Pendubot at the upright equilibrium point, we will show in Section 4 that the Pendubot can be swung up to an arbitrarily small neighborhood of the upright equilibrium point.