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Vibration analysis of functionally graded carbon nanotube reinforced composite thick plates with elastically restrained edges

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ABSTRACT

For the first time, to the authors' knowledge, the problem of the free vibration of functionally graded carbon nanotube (FG-CNT) reinforced composite moderately thick rectangular plates with edges elastically restrained against transverse displacements and rotation of the plate cross section is considered. The element-free improved moving least-squares Ritz (IMLS-Ritz) method is employed for the analysis. The first-order shear deformation theory (FSDT), accounting for transverse shear strains and rotary inertia, is used in the theoretical formulation. The applicability of the formulation is illustrated by solving a selection of example problems. The numerical results are validated through comparison and convergence studies. The effect of elastically restrained edges on the vibration behavior of the FG-CNT reinforced composite plates is studied by taking into account the CNT volume fraction ratio, CNT distribution, plate thickness-to-width ratio and plate aspect ratio.

1. Introduction

Plates are common structural components adopted in engineering applications. They are used in engineering structures as floor and foundation slabs, retaining walls, bridge decks, ship hulls and mechanical devices. The volume of literature dealing with plate vibration is extensive [1–4]. In most of these investigations, perfect boundary conditions, i.e. free, simply supported or clamped, were imposed to obtain the vibration solutions. However, the actual boundary conditions of the supports may be very different from the idealized conditions. In reality, these edge supports are elastic and can be approximated by a combination of translational and rotational restraints at the boundaries. The main objective of this paper is to present a first known free vibration solution for FG-CNT reinforced composite thick plates having elastically restrained boundaries.

CNTs have extremely high stiffness. It has been found that its strength is 100 times greater than steel. A literature survey has reported the recent advances of FG-CNT reinforced composites [5]. For structural application, the FG-CNT reinforced composites are designed to be embedded in plate components. Research on the mechanical behaviors of these structural components has attracted increasing attention. Alibeigloo and Liew [6] have presented the elasticity solution of the free vibration and bending behaviors of FG-CNT reinforced composite beams with thin piezoelectric layers using the differential quadrature method. Alibeigloo [7] has also reported the three-dimensional thermoelasticity solution of FG-CNT reinforced composite plate embedded in piezoelectric sensor and actuator layers. Lei et al. [8] have employed the kp-Ritz method to compute free vibration solutions for FG-CNT reinforced composite rectangular plates in a thermal environment. The same method has been employed by Zhang et al. [9] to study the static and dynamic behaviors of FG-CNT cylindrical panels. Wu and Li [10] have developed the finite prism method to analyze the three-dimensional free vibration of FG-CNT reinforced composite plates and laminated fiber-reinforced composite plates. Shen and Zhang [11] have studied the thermal buckling behavior of FG-CNT reinforced composite plates subjected to in-plane temperature variation. Shen [12] has employed the same methodology to study the thermal buckling and postbuckling behaviors of FG-CNT reinforced composite shells. The static and free vibration characteristics of FG-CNT reinforced composite plates have been studied by Zhu et al. [13] using the finite element method. The element-free IMLS-Ritz method has been proposed by Zhang et al. [14–16] to study

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the free vibration and buckling behaviors of FG-CNT reinforced composite triangular and skew plates.

From the known literature [5–23], we have found that no existing work has been conducted on the free vibration analysis of FG-CNT reinforced composite plates with elastically restrained edges. Studies of the effects of elastically restrained edge boundaries on the free vibration of plates have been reported mostly for isotropic cases. Gorman [24] presented a superposition method for a general solution of the free vibration of thin rectangular plates with arbitrarily distributed lateral and rotational elastic edge supports. The vibration solutions were compared for plates with perfect boundary conditions as the stiffness of elastic restraints approaches infinity. The superposition method [25] was further extended to study the free vibration analysis of rectangular thick plates with uniform elastic edge supports. Saha et al. [26] employed a variational method to study the free vibration of isotropic Mindlin plates with edges elastically restrained against rotation and translation. The elastically restrained Timoshenko beam functions are used as constitutive shape functions and the boundary conditions not satisfied remain as boundary terms in the final variational energy expression. Xiang et al. [27] studied the free vibration of rectangular Mindlin plates with edges elastically restrained against transverse displacements and rotation of the plate cross section. The polynomials and basic functions were employed as the admissible functions in the Ritz method. They have presented vibration solutions for plates with fully elastically restrained edges, and combinations of classical and elastically restrained edges. Zhou [28] proposed a set of Timoshenko beam functions as admissible functions in the Ritz method for the vibration of Mindlin plates with elastically restrained edges. The Timoshenko beam functions were constructed in the transverse deflection functions and rotation functions, which are the static solutions of a Timoshenko beam under the transverse loads of a Fourier sinusoidal series distributed along the length of the beam. Ohya et al. [29] used the superposition method to study the free vibration analysis of rectangular Mindlin plates with internal columns. The plates are considered with edge conditions having uniform lateral, rotational and torsional elastic supports. The effects of column supports on the vibration modes were presented. Wu and Yu [30] employed the pb-2 Ritz method to analyze the free vibration of rectangular plates with internal columns and elastic edge supports. The analysis was carried out using Reddy's third-order shear deformation plate theory.

This paper attempts to provide a set of vibration frequencies for FG-CNT reinforced composite plates of moderate thickness [31–40] having elastically restrained edges. Element-free methods [24–33] have their own advantages over the finite element method in terms of computational efficiency. In this study, the element-free IMLS-Ritz method [41–43] is employed. The first-order shear deformation theory (FSDT), accounting for transverse shear strains and rotary inertia, is used in the theoretical formulation. The present study considers the effects of CNT volume fraction, plate thickness-to-width ratio and plate aspect ratio on the vibration behavior of the plates under different elastic edge restraints.

2. Theoretical formulation

2.1. FG-CNT reinforced composite plates

Distributions of CNTs along the thickness direction of four types of FG-CNT reinforced composite plates of length a, width b and thickness h (refer to Fig. 1) are assumed to be

$$V_{CNT}(z) = \begin{cases} V_{CNT}^{*} & (\text{UD}) \\ \left(1 + \dfrac{2z}{h}\right)V_{CNT}^{*} & (\text{FG}-V) \\ 2\left(1 - \dfrac{2|z|}{h}\right)V_{CNT}^{*} & (\text{FG}-O) \\ 2\left(\dfrac{2|z|}{h}\right)V_{CNT}^{*} & (\text{FG}-X) \end{cases}. \tag{1}$$

where V_{CNT}^{*} denotes the CNT volume fractions. Eq. (1) describes four types of FG-CNT reinforced composite plates, of which UD represents a uniform distribution of CNTs, and the other three types of functionally graded distributions of CNTs are denoted by FG-V, FG-O and FG-X. All four types of FG-CNT reinforced composite plates are assumed to have the same mass per unit volume of CNTs.

The rule of mixture is employed to estimate the effective material properties of FG-CNT reinforced composite plates. The poly (methyl methacrylate), referred to as PMMA, is selected for the matrix [44]. The material properties of the matrix polymer and CNT are: $E^m = 2.5$ GPa, $\rho^m = 1150\ \text{kg}/m^3$, $v^m = 0.34$, $G^m = E^m/2(1 + v^m)$, $E_{11}^{CN} = 5.6466$ TPa, $E_{22}^{CN} = E_{33}^{CN} = 7.0800$ TPa and $G_{12}^{CN} = G_{13}^{CN} = 1.9445$ TPa. The (10, 10) single walled CNTs (SWCNTs) (L=9.26 nm, R=0.68 nm, h^{CN}=0.067 nm, $v_{12}^{CN} = 0.175$) are used as the reinforcements. The detailed material properties of SWCNTs used for the present analysis of the FG-CNT reinforced composite plates are selected from the simulation results reported by Shen and Zhang [11] and are tabulated in Table 1.

2.2. Total energy functional

The physical system considered is an FG-CNT reinforced composite plate with four edges elastically restrained, as shown in

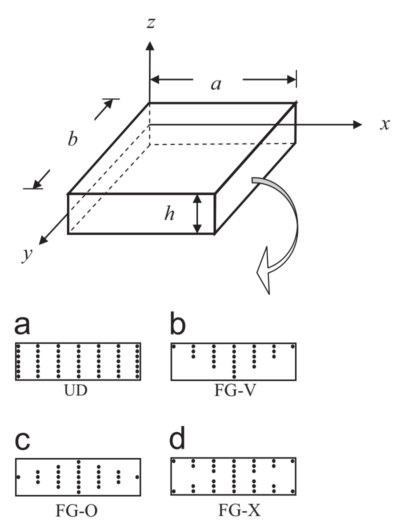

Fig. 1. Configurations of FG-CNT reinforced composite plates: (a) UD FG-CNT reinforced composite plate; (b) FG-V FG-CNT reinforced composite plate; (c) FG-O FG-CNT reinforced composite plate; and (d) FG-X FG-CNT reinforced composite plate.

Table 1
Material properties for PmPV/CNT composites reinforced by (10, 10) SWCNT under room temperature.

V_{CNT}^{*}	E_{11} (GPa)	η_1	E_{22} (GPa)	η_2	η_3
0.11	94.57	0.149	2.2	0.934	0.939
0.14	120.09	0.150	2.3	0.941	0.941
0.17	145.08	0.149	3.5	1.381	1.381