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Modeling heterogeneous materials with multiple inclusions under mixed lubrication contact

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ABSTRACT

A numerical solution for heterogeneous materials with multiple inclusions under mixed elastohydrodynamic lubrication contact is presented in this study. The pressure generated within the fluid area as well as that caused by asperity contact is obtained through solving a unified Reynolds equation system. The inclusions are modeled as homogeneous inclusions (which are assumed to have same material with matrix) with initial eigenstrains plus unknown equivalent eigenstrains by employing Eshelby's equivalent inclusion method. A modified conjugate gradient method is implemented to determine these unknowns within the governing equations. Extra deformations due to the inclusions are iteratively introduced into lubricant film thickness. The computational process is performed until the convergence of the half-space surface displacements caused by the embedded inclusions and the pressure obtained within the computational domain. This solution takes into account the interactions among the loading body, the fluid lubricant and the half-space matrix with embedded inclusions.

1. Introduction

Lubrication in mechanical power transmission systems is essential to separate the contacting surfaces and improve performance of the mechanical components. In many applications, surface roughness is often of the same order of magnitude as, or greater than, the average lubricant film thickness, so that a complete separation between the two surfaces is seldom. The lubrication regime, in which the hydrodynamic lubricant film and asperity contact coexist, is referred to mixed elastohydrodynamic lubrication (EHL) and has long been recognized. A numerical solution was firstly developed by Jiang et al. [1]. In their solution, the fluid pressure was obtained by solving the Reynolds equation while the asperity contact pressure was determined by deconvolution of contact surfaces using Fast Fourier Transform (FFT) techniques. Zhu and Hu [2] studied the mixed lubrication in point contact by utilizing a unified Reynolds equation system without identifying hydrodynamic and asperity contact regions. Afterwards, the solution was extended to line contact and plastic problems [3,4].

Generally, the materials in lubricated contact are assumed to be homogeneous. However, in practical engineering the materials are naturally heterogeneous, consisting of micro-defects such as inclusions or voids. These defects embedded beneath the contact surface would response to the external loading and result in different surface deformations than homogeneous contact. The pressure and film thickness profiles obtained based on the assumption of homogeneous materials may not be accurate enough to describe the lubricated contact.

In this study, inclusions are referred to as inhomogeneities with different elastic moduli than their surrounding material or matrix [5]. They may or may not contain eigenstrain which refers to non-elastic strain such as plastic strain and misfit strain. Inclusions and their interactions with other defects have been intensively studied [6–23]. A comprehensive survey of recent related works was provided by Zhou et al. [24]. However, few investigations have been conducted on the near-surface defects subjected to contact loading due to the intensive interactions between the loading body and heterogeneous half-space. Miller and Keer [25] investigated a two-dimensional (2D) cylindrical void or inclusion intended by a cylindrical indenter. Kuo and Chang [26] studied the stress distribution in an elastic half-space with multiple inclusions of 2D arbitrary shape when subjected to contacts.

Recently, a general study of multiple three-dimensional (3D) arbitrarily-shaped inclusions embedded in an infinite space and in a half-space was conducted by Zhou et al. [27,28] and then the same methodology was implemented to solve inclusions under contact loading [29]. Afterwards, Zhou and Wei [30] developed a semi-analytic solution for multiple subsurface cracks subject to contact loading by means of distributed dislocation technique (DDT); Dong and Zhou [31,32] and Dong et al. [33] investigated the effects of subsurface cracks and inclusions on the pressure and film thickness profiles and subsurface elastic fields.

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So far, the effects of subsurface inclusions under mixed lubricated contact have not been investigated due to intensive interactions between the fluid lubricant and the contact structures. The pressure generated within lubricants as well as that induced by solid direct contact would cause the response of the inclusions, which in return would affect the film thickness and the pressure distribution. This paper aims to develop an approach to solving this problem and provides guidance for the analysis of the heterogeneous mixed lubrication.

2. Model formulation and solution method

2.1. Problem description and solution approach

Consider the EHL system between a half-space (E_1, μ_1) sliding at the velocity v_1 and an elastic ball (E_2, μ_2) rolling at v_2. The half-space is bounded by the plane surface $z=0$ in an x–y–z Cartesian coordinate system with the lubricant flowing at an effective velocity $v_e=(v_1+v_2)/2$ (Fig. 1). The half-space contains n arbitrarily-shaped subdomains $\Omega_\psi(\psi=1,2,\ldots,n)$, each of which has material constants different from the matrix.

The domain under the contacting surfaces is discretized into $N_x \times N_y \times N_z$ cubic elements of the same size $2\Delta_x \times 2\Delta_y \times 2\Delta_z$ and each element is indexed by a sequence of three integers $[\alpha_0,\beta_0,\gamma_0](0 \le \alpha_0 \le N_x-1, 0 \le \beta_0 \le N_y-1,\ 0 \le \gamma_0 \le N_z-1)$, while the contact surface is composed of $N_x \times N_y$ square patches of $2\Delta_x \times 2\Delta_y$.

In order to formulate the governing equations, each inclusion is simulated by a homogeneous inclusion (which has the same material as the matrix) with initial eigenstrain $\varepsilon_{ij}^{\mathrm{p}}$ plus properly determined equivalent eigenstrain ε_{ij}^{*} by means of Eshelby's equivalent inclusion method (EIM). The disturbed deformation u due to the eigenstrains is then introduced into the film thickness equation. Further details on the calculation of the eigenstrains and the disturbed deformation can be found in Appendix.

The lubrication film thickness with u considered can be calculated by

$$h=h_0+\frac{x^2}{2R_x}+\frac{y^2}{2R_y}+u_z(x,y,t)+\delta_1(x,y,t)+\delta_2(x,y,t)+u(x,y,t), \qquad (1)$$

where h_0 is the initial film thickness; δ_1 and δ_2 denote the roughness amplitude of surface 1 and 2, respectively. u_z is the elastic deformation of the contact surfaces and can be obtained by

$$u_z(x,y,t)=\frac{2}{\pi E}\int\int_{\Omega}\frac{p(x',y',t)}{\sqrt{(x-x')^2+(y-y')^2}}dx'dy', \qquad (2)$$

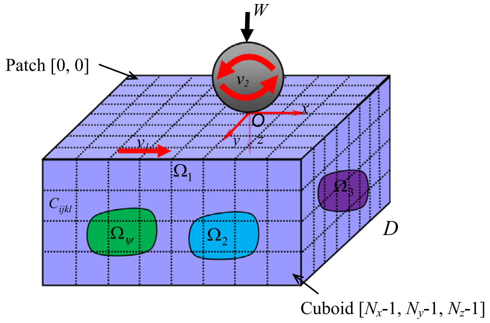

Fig. 1. Description of the mixed lubricated problem with multiple arbitrarily shaped inclusions beneath the contacting surfaces.

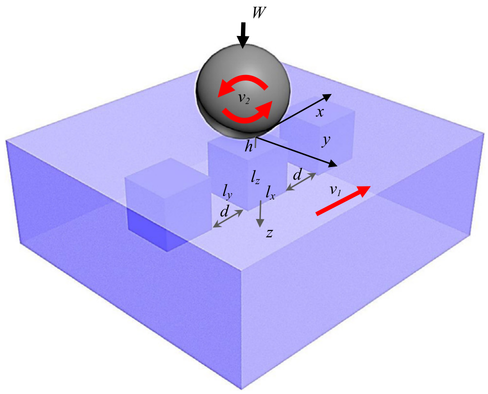

Fig. 2. A stringer of inclusions beneath the contact area.

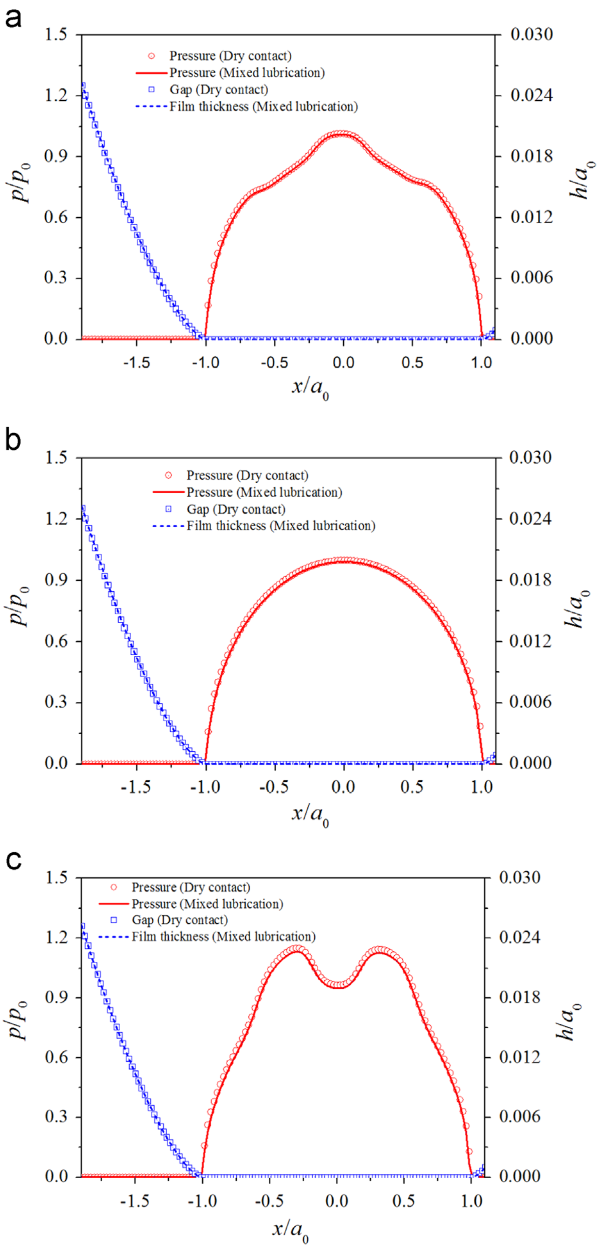

Fig. 3. Pressure distribution and film thickness profiles comparison between solutions obtained from the mixed EHL model and dry contact analysis for inclusions of (a) $\lambda=0.5$, (b) $\lambda=1.0$ and (c) $\lambda=2.0$.